CHAPTER 10: GAMES WITH ASYMMETRIC INFORMATION

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Review:
Nash Equilibrium

Discussion of the common knowledge assumption

Game 1:

<table>
<thead>
<tr>
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<th>Player 2</th>
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<tr>
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<td>a</td>
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<td>4,4</td>
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<td>b</td>
<td>5,1</td>
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<tr>
<td>c</td>
<td>2,0</td>
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Player 1

Is there a strictly (weakly) dominated strategy for each player?

Note that (c,c) is a NE strategy profile.

Which strategy would you play?
Game 2:  

<table>
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<td>a</td>
<td>9,9</td>
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<tr>
<td>b</td>
<td>10,0</td>
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<tr>
<td>c</td>
<td>4,4</td>
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The strategy profiles \((b,b)\) and \((c,c)\) are both NE strategy profiles.

What would you do if you were playing this game? Would pre-play communication help?

Game 3: Repeated game: Game 2 is played twice.

How many pure strategy profiles are NE strategy profiles?

A strategy for player \(i\) is \(s_i = (s_i^1, s_i^2(h^1))\) where \(h^1\) is the history of the game (the actions taken in stage 1 of the game.)
Define \( \bar{s}_i = (\bar{s}_i^1, \bar{s}_i^2 (h^1)) \) where \( \bar{s}_i^1 = \bar{s}_i^2 (h^1) = c \). The strategy profile \((\bar{s}_1, \bar{s}_2)\) is a NE.

What other NE are there in this two stage game?

Would players use Pareto Inferior NE strategies with pre-play communication?

How about without such communication?

Are there other pure strategy NE of this game?
Pareto efficient NE

Define \( \hat{s}_i = (\hat{s}_i^1, \hat{s}_i^2(h^1)) \) where \( \bar{s}_i^1 = a \) and \( \hat{s}_i^2(h^1) = \begin{cases} b, & h^1 = \text{opponent chooses } a \text{ in stage 1} \\ c, & h^1 = \text{opponent does not choose } a \text{ in stage 1} \end{cases} \).

The strategy profile \((\hat{s}_1, \hat{s}_2)\) is a sub-game perfect NE

After pre-play communication what would you do?
**Bayesian games**

one or more players must choose an action knowing the preferences of his opponents only probabilistically.

player \( i \in \mathcal{I} \) with utility function \( u_i(\cdot, \theta_i) \) is said to be a player of type \( \theta_i \) where

\( \theta_i \in \Theta_i \), the set of possible types of player \( i \).

Finite types: \( \Theta = \{\theta_1, ..., \theta_T\} \)

Continuous types \( \Theta = [\alpha, \beta] \)

**Definition: Common knowledge beliefs**

The joint probability distribution, \( f(\theta_1, ..., \theta_I) \), over the type space \( \Theta = \times_{i \in \mathcal{I}} \Theta_i \), is common knowledge.
Simultaneous move Bayesian game

player $i \in I$ has a type $\theta_i \in \Theta_i$ and a set of feasible actions $A_i$.

Let $S_i$ be the set of all probability measures on $\Delta(A_i)$ and define $S = S_1 \times \ldots \times S_I$. A strategy $s_i$ for player $i \in I$ is, for each type $\theta_i \in \Theta_i$, a probability measure $s_i(\theta_i) \in S_i$. The strategy profile $s = (s_1, \ldots, s_I)$ is then a listing of the type contingent strategies of every player.

Definition: Bayesian Nash equilibrium of a simultaneous move game

Let $u_i(s, \theta_i)$ $s \in S$ be the payoff of player $i \in I$ if his type is $\theta_i \in \Theta_i$ and the strategy profile is $s$. Let $f(\theta_1, \ldots, \theta_I)$ be the joint probability distribution over types, where this is common knowledge. The strategy profile is a Bayesian Nash equilibrium (BNE) if, for each $\theta_i \in \Theta_i$ and each $i \in I$, $s_i(\theta_i)$ is a best response, given the common knowledge beliefs.
**Bidding games**

Bidding games are an important class of simultaneous move Bayesian games. Each player (or bidder) has private information about the value of a prize. The players then bid for the prize by choosing actions. The resulting allocation of the prize is based on the actions chosen.

**Sealed bid auctions**

A single item is up for sale. Buyers submit bids. 

- high bid is the winning bid (tie breaking rule)

**Sealed first price (or high bid) auction**

the winning bidder pays his own high bid (the first price).

**Sealed second price auction**

the winning bidder pays the second highest bid (the second price).

**All pay auction**

All bidders submit cash bids in an envelope. The envelopes are opened and the winner is the bidder who has paid the most cash. The cash in all the envelopes is kept by the seller.
Equilibrium bidding in the sealed second price auction

Key assumption: Private values: Buyer $i$’s value is $\theta_I$

**Proposition:** In the sealed second price auction bidding $b_i = \theta_i$ is a dominant strategy.

**Proof:** Consider bidding $b_i < \theta_i$ instead of $\theta_i$. Let $m$ be the largest of the other bids. Consider the four possible cases (i) $m > \theta_i$ (ii) $m = \theta_i$ (iii) $m < b_i$ (iv) $b_i \leq m < \theta_i$

Case (i) Buyer $i$ loses regardless of whether he bids $b_i$ or $\theta_i$

Case (ii) If buyer $i$ bids $b_i$ he loses. If he bids $\theta_i$ he either loses or wins and $m = \theta_i$ so his payoff is again 0.

Case (iii) buyer $i$ wins and pays $m$ regardless of whether he bids $b_i$ or $\theta_i$

**Case (iv) buyer i loses if he bids $b_i$ and has a payoff $\theta_i - m > 0$ if he bids $\theta_i$.

Therefore bidding $b_i < \theta_i$ is a weakly dominated strategy.

Exercise: Argue that bidding $b_i > \theta_i$ is also a weakly dominated strategy.
Equilibrium payment in the sealed second price auction

Let \( \{\theta_1, \theta_2, \ldots, \theta_n\} \) be the values ordered from highest to lowest.\(^1\) If all buyers bid according to their dominant strategies and buyer \( i \) wins, then \( \theta_i = \theta_1 \) and his payment is \( \theta_2 \). Buyer \( i \)'s expected payment conditional upon being the winner is therefore \( E\{\theta_2 | \theta_1 = \theta_i\} \).

**Assumption:** i.i.d values joint probability density \( f(\theta_1) \times \ldots \times f(\theta_n) \), \( \theta_i \) has c.d.f. \( F(\theta_i) \)

\[
\Pr\{\text{bidder } j \text{'s value is less than } \theta | \text{bidder } i \text{'s value is } \theta_i\} = \frac{F(\theta)}{F(\theta_i)}
\]

\[
\Pr\{\text{all other } n-1 \text{ bidders values are less than } \theta | \text{bidder } i \text{'s value is } \theta_i\} = \frac{F_{n-1}(\theta)}{F_{n-1}(\theta_i)}
\]

\[
E\{\theta_2 | \theta_1 = \theta_i\} = \frac{\int_{0}^{\theta_i} \theta dF_{n-1}(\theta)}{F_{n-1}(\theta_i)}
\]

\(^1\) The \( j \) th highest value is called the \( j \) th order statistic
Sealed first price (high bid) auction

**Assumption:** i.i.d values joint probability density $f(\theta_1) \times \cdots \times f(\theta_n)$, $\theta_i$ has c.d.f. $F(\theta_i)$

Symmetric equilibrium $b_i = B(\theta_i)$ Assume $B(\theta)$ is a strictly increasing continuous function.

Fig. 10.2-1: Equilibrium bid function
Define $w(\theta_i)$ to be buyer $i$’s equilibrium win probability if he bids $B(\theta_i)$.

If all other buyers bidding according to the equilibrium bidding strategy,

$$w(\theta_i) = F^{n-1}(\theta_i).$$

The equilibrium (expected payoff) is

$$V(\theta) = w(\theta)(\theta - B(\theta)).$$

$$= \text{Pr}\{B(\theta) \text{ is winning bid}\} (\text{net gain if buyer wins})$$

Suppose that buyer $i$ deviates from the equilibrium strategy $B(\theta)$ and bids $B(\theta_i)$ when his value is $\theta \neq \theta_i$. If his value is $\theta$, then his expected payoff is

$$u(\theta) = w(\theta_i)(\theta - B(\theta_i)).$$

Note that the expected payoff $u(\theta)$ rises linearly with buyer $i$’s value.

Note also that $V(\theta_i) = u(\theta_i)$

It must also be true that for all $\theta \neq \theta_i$, $V(\theta) > u(\theta)$
Recap

Equilibrium (expected payoff)

\[ V(\theta) = w(\theta)(\theta - B(\theta)). \]

Payoff to deviation

\[ u(\theta) = w(\theta_j)(\theta - B(\theta_j)). \]

Buyer \( i \) with value \( \theta \) bids

\( B(\theta_j) \) rather than \( B(\theta) \)

Therefore the functions \( V(\theta) \)

and \( u(\theta) \) must have the same

slope at \( \theta_i \). Hence

\[ \frac{dV}{d\theta}(\theta_i) = \frac{du}{d\theta}(\theta_i) = w(\theta_i) \]

\[ V(\theta_i) = V(\theta_i) - V(0) = \int_0^{\theta_i} \frac{dV}{d\theta} d\theta = \int_0^{\theta_i} w(\theta) d\theta. \]
\[ V(\theta_i) = V(\theta_i) - V(0) = \int_0^\theta \frac{dV}{d\theta} d\theta = \int_0^\theta w(\theta) d\theta. \]

Integrating this expression by parts,

\[ V(\theta_i) = \theta_i w(\theta_i) - \int_0^\theta \theta w'(\theta) d\theta = w(\theta_i)(\theta_i - \frac{\int_0^\theta \theta dw(\theta)}{w(\theta_i)}) - \frac{\int_0^\theta \theta dF^{n-1}(\theta)}{F^{n-1}(\theta)} \]

Also \[ V(\theta_i) = w(\theta_i)(\theta - B(\theta_i)). \]

\[ B(\theta_i) = \frac{\int_0^\theta \theta dF^{n-1}(\theta)}{F^{n-1}(\theta)} \]

(compare with result on page 9)

Therefore the equilibrium bid is

\[ B(\theta_i) = E\{\theta_2 \mid \theta_1 = \theta_i\} \]
Buyer $i$ thus bids the expectation of the second highest value, conditional upon buyer $i$ having the highest value. As we have seen, this is also the equilibrium expected payment of the high value buyer in the sealed second price auction. We therefore have the following proposition.

**Proposition: Revenue equivalence of the sealed first and second price auctions**

In an $n$-bidder auction in which bidders are risk neutral and valuations are independently and identically distributed according to a distribution with c.d.f. $F \in C^1$ and support $[0, \beta]$, equilibrium expected revenue is the same in the sealed first and second price auctions.
Understanding the revenue equivalence theorem

Reconsider the sealed second price auction.

Let $V(\theta)$ be the equilibrium expected payoff.

If buyers bid according to their equilibrium strategies, buyer $i$ with value $\theta_i$ will bid $b_i = \theta_i$, so win probability $w(\theta_i) = F^{n-1}(\theta_i)$.

Let $r_i$ be buyer $i$’s expected payment.

Then buyer $i$’s expected payoff when his value is $\theta_i$ is

$$V(\theta_i) = w(\theta_i)\theta_i - r_i.$$ 

Deviation by buyer $i$: bid $\theta_i$ regardless of his value.

Then his payoff is

$$u(\theta) = w(\theta_i)\theta - r_i.$$ 

Since this strategy is feasible and $V(\theta)$ is the best response payoff it follows that

$$V(\theta) \geq u(\theta) = w(\theta_i)\theta - r_i.$$ 

Also $V(\theta_i) = w(\theta_i)\theta_i - r_i$. 
Thus the equilibrium payoff function is bounded below by the line \( u(\theta) = w(\theta_i)\theta - r_i \) and touches at \( \theta = \theta_i \) as depicted below.

It follows that the slope of the equilibrium payoff function is \( \frac{dV}{d\theta}(\theta_i) = w(\theta_i) \).

Fig. 10.1-3: Equilibrium payoffs in the sealed second price auction
Therefore just as in the sealed first price auction

\[ V(\theta_i) = V(\theta_i) - V(0) = \int_0^{\theta_i} \frac{dV}{d\theta} d\theta = \int_0^{\theta_i} w(\theta) d\theta. \]

We therefore have the following proposition.

**Proposition: Buyer equivalence of the sealed first and second price auctions**

In an \( n \)-bidder auction in which bidders are risk neutral and valuations are independently and identically distributed according to a distribution with c.d.f. \( F \in C^1 \) and support \([0, \beta]\), the equilibrium payoff for each buyer type the same in the sealed first and second price auctions.

In both of the auctions the total surplus generated is the value of the transfer of the item from the seller to the buyer who values it the most. Therefore the total surplus in the two auctions is the same. In equilibrium, the buyers are indifferent between the two auctions therefore the remaining player in the game (the seller) must be as well. Hence we have revenue equivalence.
**Another angle**

Define

\[ U(\theta_i, x) = w(x)(\theta - B(x)) \]

\[ U_\theta(\theta_i, x) = \frac{\partial U}{\partial \theta}(\theta_i, x) = w(x) \]

\[ U_x(\theta_i, x) = \frac{\partial U}{\partial x}(\theta_i, x) = w'(x)\theta_i - \frac{d}{dx}w(x)B(x) \]

But \( x = \theta_i \) is optimal so \( U_x(\theta_i, \theta_i) = 0 \).

\( V(\theta) = U(\theta, \theta) \).

Then

\[ \frac{dV}{d\theta} = U_\theta(\theta_i, \theta_i) = w(\theta_i) \]

**Remark: Understanding the Revelation Principle**
Strategically equivalent open auctions

Dutch Auction

**Proposition: Equivalence of the sealed high-bid and Dutch auctions**

The equilibrium bidding strategies in the sealed high-bid and Dutch auctions are identical.

Ascending bid (English) auction

**Proposition: Equivalence of the sealed second price and English Auctions**

The equilibrium bidding strategy in a sealed second price and in the open ascending-price auction is for buyers to bid their values.
All pay auction

Let $B(\theta_i)$ be the equilibrium bid function.

If all buyers bid according to their equilibrium bid functions and buyer $i$’s value is $\theta$, buyer $i$’s win probability is $w(\theta) = F^{n-1}(\theta)$ and so her equilibrium payoff is

$$V(\theta) = w(\theta)\theta - B(\theta)$$

Suppose that she deviates and bids $B(\theta_i)$ when her value is $\theta$.

(This must be sub-optimal for $\theta \neq \theta_i$ since $B(\theta)$ is her best response.

Her payoff from deviating is

$$u(\theta) = w(\theta_i)\theta - B(\theta)$$
Multiple items for sale

Example: 2 identical items and 3 buyers. Each winner pays his bid.

Suppose you are buyer 1. Both of the other buyers have a lower value than you with probability

$$\Pr\{\theta_2 \leq \theta_1\} \Pr\{\theta_3 \leq \theta_1\} = F^2(\theta_1)$$

You have a higher value than 2 and a lower value than 3 with probability

$$\Pr\{\theta_2 \leq \theta_1\} \Pr\{\theta_3 \geq \theta_1\} = F(\theta_1)(1 - F(\theta_1))$$

You have a higher value than 3 and a lower value than 2 with probability

$$\Pr\{\theta_3 \leq \theta_1\} \Pr\{\theta_2 \geq \theta_1\} = F(\theta_1)(1 - F(\theta_1)).$$

Thus your win probability is

$$w(\theta_1)F^2(\theta_1) + 2F(\theta_1)(1 - F(\theta_1)) = 2F(\theta_1) - F^2(\theta_1)$$

Now follow the usual steps to solve for the equilibrium bid function.

Would winners prefer to both pay the second bid?
Sequential Move Games with incomplete information

\( i_t \in I \) is the player who moves in the \( t \)-th stage.

The set of possible pure strategies available to player \( i_t \) in stage \( t \) is \( A_i \).

\( S_t \) is the set of probability measures on \( \Delta(A_i) \).

The stage \( t \) strategy \( s_i(h^{t-1}, \theta_t) \in S_i \) is a probability measure for each possible history and type.

A payoff \( u_i(s, \theta_t), \ i \in I \) is specified for every possible strategy profile \( s \).

**Definition: BNE of a sequential move game**

Let \( u_i(s, \theta_t) \) \( s \in S \) be the payoff of player \( i \in I \) if his type is \( \theta_i \in \Theta_i \). Let \( f(\theta_1, \ldots, \theta_i) \) be the joint probability distribution over types, where this is common knowledge. A strategy profile is a BNE if, for each \( t \) and \( \theta_{i_t} \in \Theta_{i_t} \), \( s_t(h^{t-1}, \theta_{i_t}) \) is a best response, given the common knowledge beliefs.
Refinements of Bayesian Nash Equilibrium

Example: Entry Game

player 1, the (potential) Entrant and player 2, the Incumbent must choose a strategy before knowing how strong player 1’s financial backing will be.\(^1\) If player 1’s backing is weak, the payoff matrix is as shown below.

\[
\begin{array}{c|cc}
& \text{Fight} & \text{Share} \\
\hline
\text{Enter} & -2,40 & 3,30 \\
\text{Out} & 0, 60 & 0, 60 \\
\end{array}
\]

Fig 10.2-1a: Weak Entrant and Incumbent

\(^1\) Player 1 (the Entrant) is male and player 2 (the Incumbent) is female.
If player 1’s financial backing is strong the payoff matrix is instead as follows.

\[
\begin{array}{c|cc}
\text{Player 2:} & \text{Fight} & \text{Share} \\
(\text{Incumbent}) & & \\
\hline
\text{Player 1:} & \text{Enter} & 4,29 & 3,30 \\
(\text{Entrant}) & \text{Out} & 0,60 & 0,60 \\
\end{array}
\]

Fig. 10.2-1b: Strong Entrant and Incumbent

The probability that the Entrant is weak is \( p_w < 1/11 \). This is common knowledge.
Full information NE

1. Weak Entrant

   \((Out, \text{Fight})\) is the unique NE strategy profile.

2. Strong Entrant

   \((Out, \text{Fight})\) and \((\text{Enter}, \text{Share})\) are both NE strategy profiles.

Only one is sub-game perfect
Perfect Bayesian Equilibrium

Now consider the game when neither player knows player 1’s type (remember $p_w < 1/11$)

Fig. 10.2-3 Game with both players uninformed
BNE with entry

Strategy profile \((\text{Enter, Share})\)

With player 2 choosing \(\text{Share}\), player 1’s best response is \(\text{Enter}\).

Given entry by player 1

\[
U_2(\text{Share}) = 30, \quad U_2(\text{Fight}) = p_w 40 + (1-p_w)29 = 29 + 11p_w
\]

\[
U_2(\text{Fight}) - U_2(\text{Share}) = 11(p_w - \frac{1}{11}) < 0
\]

BNE with no entry

With player 2 choosing \(\text{Fight}\), if player 1 chooses \(\text{Enter}\) his payoff is

\[
U_1 = p_w u_{1w}(\text{Enter, Fight}) + (1-p_w)u_{1s}(\text{Enter, Fight}) = p_w (-2) + (1-p_w)(-1) = -1 - p_w < 0.
\]

Since the payoff to choosing \(\text{Out}\) is zero, player 1’s best response is to choose \(\text{Out}\).
Is one of these equilibria more plausible than the other?

(i) continuity argument for sufficiently small $p_w$

(ii) for $p_w < \frac{1}{11}$ the best response at player 2’s decision node is *Share* not *Fight*.

**Beliefs on and off the equilibrium path**

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**Definition: Perfect Bayesian Equilibrium**

A BNE strategy profile is a perfect Bayesian equilibrium (PBE) if, at all nodes and information sets off the equilibrium path, there are strategies and beliefs consistent with Bayes’ Rule such that the strategies at these nodes and information sets are best responses.

---

Fight is a best response only if $Pr\{\text{weak} | \text{Enter}\} \geq \frac{1}{11}$. But player 1 does not know his type. Thus there is no information that could lead to a revision of player 2’s beliefs about which player has entered. Thus player 1 must believe that $p_w < \frac{1}{11}$.

Thus (*Out, Fight*) is not a PBE.
Sequential PBE

Suppose that the $t$-th stage action set is $A_t$, where $A_t$ is finite. Let $\bar{s} \in S_t$, $t = 1, \ldots, T$ be the stage $t$ BNE strategy (a probability measure on $\Delta(A_t)$). Then the BNE strategy profile is $\bar{s} = (\bar{s}_1 \times \ldots \times \bar{s}_T)$. A strategy $s_t$ is called “completely mixed” if it places positive probability on all actions. Then consider sequences $\{s^k\}_{k=1}^{\infty}$ of completely mixed strategies that approaches $\bar{s}$ as $k \to \infty$. For each $k$ it is possible to compute the conditional probability of the nodes in any information set off the equilibrium path. The limiting conditional probabilities of these nodes are the sequential conditional probabilities. Having computed the sequential conditional probabilities, the final step is to check whether the proposed strategy at every information set (or node) off the equilibrium path is a best response.

**Definition: Sequential PBE (Kreps)**

A PBE strategy profile of a game is sequential if the probability of every node off the equilibrium path is computed as the limit of probabilities implied by trembles, when the probability of all such trembles approaches zero.
A stronger variant of the sequential PBE is the trembling hand PBE. Just as in the sequential PBE, let \( \bar{s} \) be a BNE strategy profile and consider a sequence \( \{s^k\}_{k=1}^\infty \) of completely mixed strategies that approaches \( \bar{s} \) as \( k \to \infty \). A necessary condition for a trembling hand PBE is that the strategies at each information set are best responses given the limiting beliefs. Thus a trembling hand PBE is sequential. In addition, the BNE strategies must remain best responses when the trembles are sufficiently small. Formally, there must be some \( \hat{k} \) such that, for all \( k > \hat{k} \), the BNE strategies are best responses given the beliefs at each decision node and information set off the equilibrium path implied by the completely mixed strategy profile \( \pi^k \).

**Definition: Trembling-hand perfect equilibrium (Selton)**

A BNE is a trembling hand PBE if there exists some sequence of completely mixed strategy profiles, \( \{s^k\}_{k=1}^\infty \), converging on the equilibrium strategy profile, such that for all sufficiently large \( k \), the equilibrium strategies are best responses at every decision node and information set off the equilibrium path.

---

2 Exercise 10.2-2 provides an example where a “crazy” equilibrium is sequential but not trembling hand perfect. As a practical matter it is often easier to check whether an equilibrium is sequential so this should be the first step.
Sequential move games with private information

Example 2: A new entry game.

BNE strategy profiles

(i) \( s_1(\text{weak}) = \text{Out} \), \( s_1(\text{strong}) = \text{Enter} \),
\( s_2(h^l = \text{Enter}) = \text{Share} \)

(ii) \( s_1(\text{weak}) = \text{Out} \), \( s_1(\text{strong}) = \text{Out} \),
\( s_2(h^l = \text{Enter}) = \text{Fight} \)

Both are sequential PBE

BNE (i): the conditions for a sequential PBE are satisfied vacuously.

Fig. 10.2-6: Game with an informed Entrant
BNE (ii): To compute sequential beliefs we introduce trembling probabilities $\varepsilon_w$ and $\varepsilon_s$ for the two types of player 1. The conditional probability that the entrant is weak is

$$\Pr\{W \mid Enter\} = \frac{\Pr\{\text{Weak, Enter}\}}{\Pr\{\text{Weak, Enter}\} + \Pr\{\text{Strong, Enter}\}}$$

$$= \frac{p_w \varepsilon_w}{p_w \varepsilon_w + (1 - p_w) \varepsilon_s}.$$

Choose $\varepsilon_w = \varepsilon$ and $\varepsilon_s = \varepsilon^2$

Then

$$\Pr\{W \mid Enter\} = \frac{p_w \varepsilon}{p_w \varepsilon + (1 - p_w) \varepsilon^2} = \frac{p_w}{p_w + (1 - p_w) \varepsilon} \rightarrow 1 \text{ as } \varepsilon \rightarrow 0.$$  

Given this belief player 2’s best response is *Fight.*

Fig. 10.2-6: Game with an informed Entrant
The Intuitive Criterion
Consider the equilibrium in which both types of player 1 choose Out. If player 1 is strong, he might make the following argument to himself or to player 2. “If I were weak and were to choose Enter, my possible payoffs would be –1 and –6, while my equilibrium payoff is zero. Thus I would never choose Enter. Therefore if I choose Enter, I will be credibly signaling to player 2 that I am strong. Her best response will then be Share. As a result, I will end up with a payoff of 1. Since this is better than staying out I should choose to Enter.”
Any PBE in which a particular type can make such an argument is said to fail the Intuitive Criterion.

Cho and Kreps’ Intuitive Criterion
Let \( \hat{a}_i \) be a pure strategy of player \( i \), that is chosen with zero probability in a PBE. Let \( u_i(\hat{a}_i, \theta_i) \) be player \( i \)’s payoff if he chooses \( \hat{a}_i \) and is believed to be type \( \theta_i \in \Theta_i \). Let \( u_i^N(\theta_i) \) be this type’s PBE payoff. The PBE fails the Intuitive Criterion if, for some player \( i \) of type \( \hat{\theta}_i \in \Theta_i \), \( u_i(\hat{a}_i, \hat{\theta}_i) > u_i^N(\hat{\theta}_i) \) and for all other types in \( \Theta_i \), \( u_i(\hat{a}_i, \theta_i) < u_i^N(\theta_i) \).

Example 2: PBE in which both types choose Out fails the Intuitive Criterion.