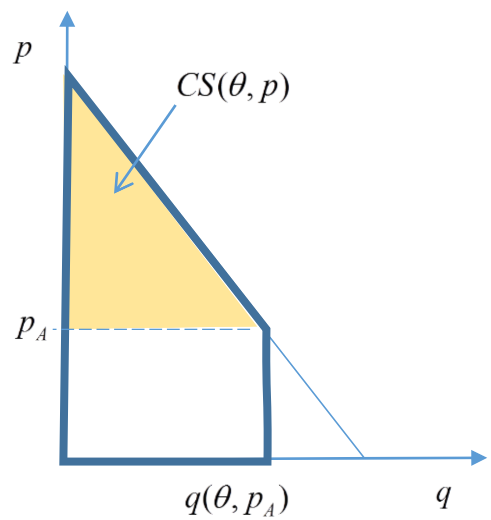
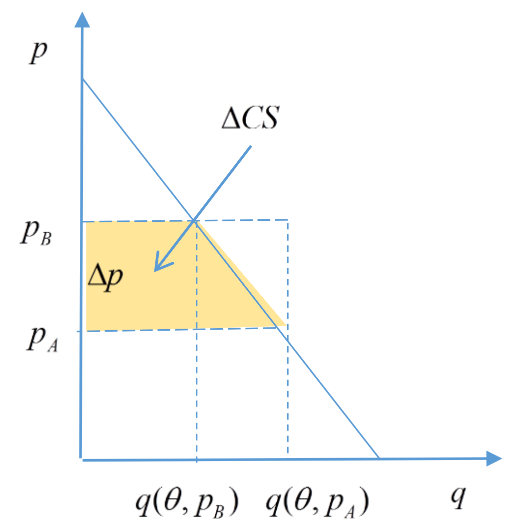
Direct Price Discrimination with two part pricing

The area under the demand price function is the total benefit $B(\theta,q)$ where θ is a parameter.

Examples: $(i)\ B(\theta,q) = 2\theta q - q^2$, $(ii)\ B(\theta,q) = \theta q - \dfrac{q^2}{1+\theta}$

The net gain to the consumer after paying the price per unit p_A (the "consumer surplus") is the area of the shaded region.





Suppose that the price rises from p_A to p_B . The reduction in consumer surplus is the heavily shaded region.

From the figure, this lies between

$$-q(\theta, p_A)(p_B - p_A) \text{ and } -q(\theta, p_B)(p_B - p_B).$$

That is

$$-q(\theta, p_A)(p_B - p_A) \le \Delta CS \le -q(\theta, p_B)(p_B - p_A)$$

Then

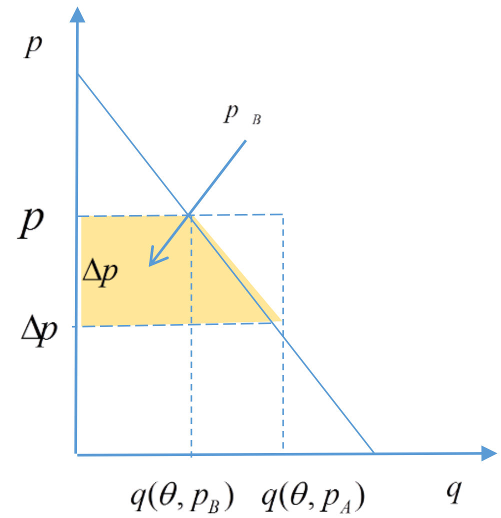
$$-q(\theta, p_A) \le \frac{\Delta CS}{\Delta p} \le -q(\theta, p_B)$$

Taking the limit as $p_B \to p_A$,

$$-q(\theta, p_A) \le \frac{dCS}{dp} \le -q(\theta, p_A).$$

Therefore

$$\frac{d}{dp} CS(\theta, p_A) = q(\theta, p_A)$$

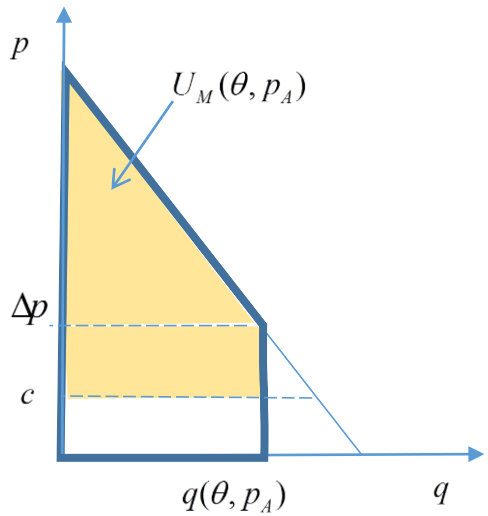


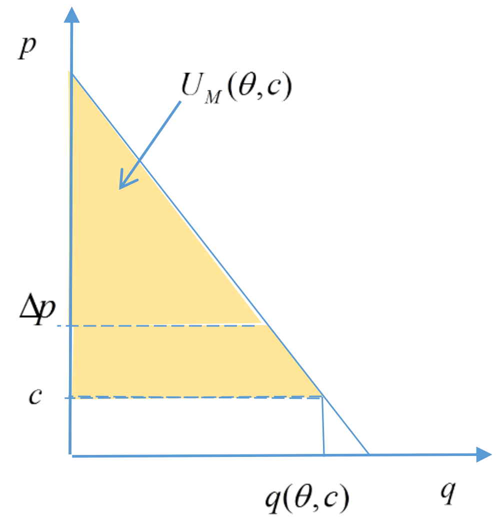
With two part pricing the monopoly can extract all the consumer surplus by charging an access fee equal to this surplus.

$K(\theta) = \mathrm{CS}(\theta, p)$.

Profit:

$U_M(p) = B(\theta, q(\theta, p)) - cq(\theta, p)$





maximized by setting the price equal to marginal cost.

The consumer surplus is $CS(\theta, c)$.

The monopoly covers its costs with the use fee, $p = c$

and charges an access fee K equal to the consumer surplus.

Indirect price discrimination with two part pricing

For direct price discrimination

(i) resale is prohibitively costly

(ii) the monopoly can identify the different types of buyer

(iii) there are no legal constraints to excluding different classes of customer from offers.

But what if either assumption (ii) or assumption (iii) is not satisfied.

T different types of customer. We label them so that higher types have higher demands. Formally,

Assumption 1: Higher types have higher demand functions

For all θ_s and $\theta_t > \theta_s$ if $p(\theta_s, q) \geq 0$ then $p(\theta_t, q) > p(\theta_s, q)$

For simplicity we will focus on linear demand price functions.

$$q = (a(\theta) - p) / b(\theta), \ \theta \in \{\theta_1, \theta_2, \ldots\}$$

Suppose that the monopoly offers a set S of n alternative plans

$$S = \{(p_a, K_a), \ldots, (p_n, K_n)\}.$$

Note that the alternative $(p_0, K_0) = (a_T, 0)$ is equivalent to purchasing nothing.

augmented set

$$S^0 = \{(p_0, K_0), (p_a, K_a), \ldots, (p_n, K_n)\}$$

Next define $(p(\theta_s), K(\theta_s))$ to be the **choice** of type θ_s from this set of plans. We will write the set of choices as

$$C = \{(p(\theta_1), K(\theta_1), \ldots, (p(\theta_T), K(\theta_T))\}.$$

Since $(p(\theta_s), K(\theta_s))$ is the choice of a type θ_s buyer,

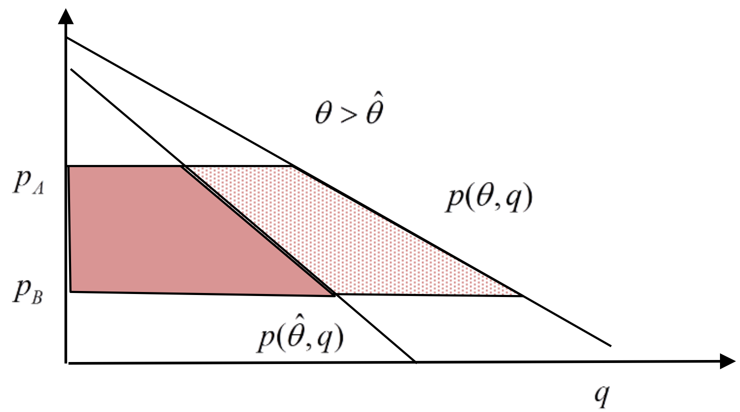
$$u(\theta_s, p(\theta_s), K(\theta_s)) \geq u(\theta_s, p(\theta_t), K(\theta_t)), \text{ for all } \theta_t$$ in the set of choices.

Key To the analysis: *Higher types are willing to accept a higher access fee in return for a lower price per unit.*

Consider two plans (p_A, K_A) and (p_B, K_B) where $p_B < p_A$.

The increase in consumer surplus if switching to the second plan is

$$CS(\theta, p_B) - CS(\theta, p_A)$$



This is depicted in the Figure. Higher types have higher demand price functions. Thus the increase in consumer surplus is greater for higher types.

We now argue that higher types will never choose a plan with a higher use fee.

Principle 1: Monotone choices

Let $(p(\theta_s), K(\theta_s))$ and $(p(\theta_t), K(\theta_t))$ be the choices of type θ_s and type $\theta_t > \theta_s$.

Then $p(\theta_t) \le p(\theta_s)$ and $K(\theta_t) \ge K(\theta_s)$

Consider any plan (p_A, K_A) where $p_A > p(\theta_s)$. Since the latter is the choice of type θ_s this type of customer has revealed that he was better off paying the higher access fee and getting the lower unit price. That is, his increase in consumer surplus outweighed the difference in access fees. All higher types have a greater increase in consumer surplus when switching to a plan with a lower price so they must strictly prefer $(p(\theta_s), K(\theta_s))$ over (p_A, K_A) .

We have therefore shown that no higher type will purchase a plan with a higher unit price that $p(\theta_s)$

We can strengthen this result to obtain the second key principle.

Principle 2:

If type $\hat{\theta}$ is indifferent between 2 plans (p_A, K_A) and (p_B, K_B) and $p_A > p_B$ then (i) all higher types strictly prefer (p_B, K_B) and (ii) all lower types strictly prefer (p_A, K_A).

The argument is almost the same.

If type $\hat{\theta}$ switches from plan A to plan B his change in payoff is

$$u(\hat{\theta}, p_B, K_B) - u(\hat{\theta}, p_A, K_A) = [CS(\hat{\theta}, p_B) - K_B] - [CS(\hat{\theta}, p_A) - K_A]$$

$$= CS(\hat{\theta}, p_B) - CS(\hat{\theta}, p_A) - K_B + K_A$$

By hypothesis this type is indifferent so the increase in consumer surplus exactly offsets the increase in the access fee.

$$u(\hat{\theta}, p_B, K_B) - u(\hat{\theta}, p_A, K_A) = CS(\hat{\theta}, p_B) - CS(\hat{\theta}, p_A) - K_B + K_A = 0$$

Repeating,

$$u(\hat{\theta}, p_B, K_B) - u(\hat{\theta}, p_A, K_A) = CS(\hat{\theta}, p_B) - CS(\hat{\theta}, p_A) - K_B + K_A = 0$$

Higher types have higher increases in consumer surplus when switching to a plan with a lower unit price so

$$u(\theta, p_B, K_B) - u(\theta, p_A, K_A) = CS(\theta, p_B) - CS(\theta, p_A) - K_B + K_A > 0 \text{ for all } \theta > \hat{\theta}$$

Lower types have smaller increases in consumer surplus when switching to a plan with a lower unit price so

$$u(\theta, p_B, K_B) - u(\theta, p_A, K_A) = CS(\theta, p_B) - CS(\theta, p_A) - K_B + K_A < 0 \text{ for all } \theta < \hat{\theta} \text{ .}$$

Profit maximizing plans

Participation Constraints

The type 1 buyer cannot be forced to participate so we require that

$$U(\theta_1) = \mathrm{CS}(\theta_1, p(\theta_1)) - K(\theta_1) \geq 0 \tag{3.1}$$

Since a higher type has a higher consumer surplus if he chooses plan 1 it follows that

$$\mathrm{CS}(\theta_2, p(\theta_1)) - K(\theta_1) \geq \mathrm{CS}(\theta_1, p(\theta_1)) - K(\theta_1) \geq 0$$

So if (3.1) holds, then the higher type is not being forced to participate either.

Revealed preference constraints (incentive constraints)

1. **No gain to switching to the plan with the closest higher use fee**

$$u(\theta_t, p(\theta_t), K(\theta_t)) = \mathrm{CS}(\theta_t, p(\theta_t)) - K(\theta_t)$$
$$\geq \mathrm{CS}(\theta_t, p(\theta_{t-1})) - K(\theta_{t-1}) = u(\theta_t, p(\theta_{t-1}), K(\theta_{t-1})) \qquad (3.2)$$

2. **No gain to switching to any other plan with a higher use fee.**

$$u(\theta_t, p(\theta_t), K(\theta_t)) = \mathrm{CS}(\theta_t, p(\theta_t)) - K(\theta_t)$$
$$\geq \mathrm{CS}(\theta_t, p(\theta_s)) - K(\theta_s) = u(\theta_t, p(\theta_s), K(\theta_s)) \text{ for all } \theta_s < \theta_t$$

3. **No gain to switching to a plan with a lower use fee**

$$u(\theta_t, p(\theta_t), K(\theta_t)) = \mathrm{CS}(\theta_t, p(\theta_t)) - K(\theta_t)$$
$$\geq \mathrm{CS}(\theta_t, p(\theta_s)) - K(\theta_s) = u(\theta_t, p(\theta_s), K(\theta_s)) \text{ for all } \theta_s > \theta_t \quad .$$

Approach: Solve a "relaxed" problem

1. Fix the prices for the plans to satisfy monotonicity:

   $p(\theta_1) \geq \ldots \geq p(\theta_T)$

2. Ignore all but the local revealed preference constraint and choose the access fees to maximize revenue.
3. Then show that the last constraint is satisfied.

Consider the two type case.

The constraints for the relaxed problem ((3.1) and (3.2)) can be rewritten as follows:

$$\mathrm{CS}(\theta_1, p(\theta_1)) \geq K(\theta_1)$$

$$\mathrm{CS}(\theta_2, p(\theta_2)) - \mathrm{CS}(\theta_2, p(\theta_1)) \geq K(\theta_2) - K(\theta_1)$$

If the first constraint is binding we can increase $K(\theta_1)$ and $K(\theta_2)$ by the same amount (so the second constraint still holds) until the first constraint is binding. Then increase $K(\theta_2)$ until the second constraint is binding.

Appealing to Principle 2, since type θ_2 is indifferent between the two plans, the lower type strictly prefers the plan with the higher price and lower access fee. Thus the other revealed reference constraint is satisfied.

Principle 3: The profit maximizing monopolist chooses $K(\theta_1) = CS(\theta_1, p(\theta_1))$ so that the payoff of a type θ_1 customer is zero.

Principle 4: With two types the monopolist chooses $K(\theta_2)$ so that a type θ_2 customer is indifferent between plan 2 and plan 1, that is $CS(\theta_2, p(\theta_2)) - K(\theta_2) = CS(\theta_2, p(\theta_1)) - K(\theta_1)$

We have therefore shown that the revenue maximizing access fees are

$$K(\theta_1) = CS(\theta_1, p(\theta_1))$$

$$K(\theta_2) = K(\theta_1) + CS(\theta_2, p(\theta_1)) - CS(\theta_1, p(\theta_1))$$

Given these principles we can use Solver to solve for the profit maximizing plans.

Total revenue from plan *t* is

$$r(\theta_t) = K(\theta_t) + p(\theta_t)q(\theta_1, p(\theta_t))$$

Total cost is $C(\theta_t) = cq(\theta_t, p(\theta_t))$

Let $n(\theta_t)$ be the number of customers of type θ_t . Then the monopoly chooses the prices to maximizes total profit

$$U_M = n(\theta_1)[r(\theta_1) - C(\theta_1)] + n(\theta_2)[r(\theta_2) - C(\theta_2)]$$

D8 =MAX(0,(C3-B8)/D3)

	A	B	C	D	E	F	G	H	I	J	K
1			Parameters								
2		type	a	b	c	n					
3		1	15	1	10	20					
4		2	30	1	10	20			TOTALS		
5									Revenue	Cost	Profit
6									8000	4000	4000
7	t	p(θt)	p(θt-1)	q(θt)	p*q	CS(θ,p)	CS(θt,pt-1)	K	Revenue	Cost	Um
8	1	36.40		0.00	0.00	0.00		0.00	0.00	0.00	0.00
9	2	10.00	36.40	20.00	200.00	200.00	0.00	200.00	400.00	200.00	200.00
10											
11											

You should download the spread-sheet TwoPartPricing.xlsm and carefully review all the formulas. Note the formula for demand. If the price is so high that $q = (q - p)/b < 0$ the demand is actually zero. Thus MAX(0,(q-p)/b) selects the correct quantity.

Similarly with consumer surplus, the quadratic formula is only correct for $p \leq a$. Therefore the correct formula is

=0.5*(MAX(0,C$3-B8))^2/D3

With two types there is a single constraint. The price for plan 2 must be less than or equal to the price for plans 1.

D8 =MAX(0,(C3-B8)/D3)

	A	B	C	D	E	F	G	H	I	J	K
1			Parameters								
2		type	a	b	c	n					
3		1	15	1	10	20					
4		2	30	1	10	20			TOTALS		
5									Revenue	Cost	Profit
6									8000	4000	4000
7	t	p(θt)	p(θt-1)	q(θt)	p*q	CS(θ,p)	CS(θt,pt-1)	K	Revenue	Cost	Um
8	1	36.40		0.00	0.00	0.00		0.00	0.00	0.00	0.00
9	2	10.00	36.40	20.00	200.00	200.00	0.00	200.00	400.00	200.00	200.00
10											
11											

The spread-sheet above depicts the solution. Note that a type θ_2 customer has a use fee equal to marginal cost and therefore consumes exactly the same as with direct price discrimination. However a type θ_1 customer has a use fee so high that he makes no purchase. All the consumer surplus can then be squeezed for high types.

In the next figure the intercept parameter for type θ_1 is higher. (This is the reservation price.)

Now the profit maximizing strategy is to have two plans.

F8 =0.5*(MAX(0,C3-B8))^2/D3

	A	B	C	D	E	F	G	H	I	J	K
1			Parameters								
2		type	a	b	c	n					
3		1	26	1	10	20					
4		2	30	1	10	20			TOTALS		
5									Revenue	Cost	Profit
6									11680	6400.001	5280
7	t	p(θt)	p(θt-1)	q(θt)	p*q	CS(θ,p)	CS(θt,pt-1)	K	Revenue	Cost	Um
8	1	14.00		12.00	168.00	72.00		72.00	240.00	120.00	120.00
9	2	10.00	14.00	20.00	200.00	200.00	128.00	144.00	344.00	200.00	144.00
10											

END OF TUESDAY LECTURE

Three or more types of customer

The analysis of the two type case can be extended directly if there are three or more types of buyer. Again we consider the relaxed problem and only consider the following three "local downward" constraints:

Participation constraint

$$U(\theta_1) = \mathrm{CS}(\theta_1, p(\theta_1)) - K(\theta_1) \geq 0 \tag{3.3}$$

No gain to switching to a plan with a higher use fee

$$\begin{aligned} u(\theta_{t+1}, p(\theta_{t+1}), K(\theta_{t+1})) &= \mathrm{CS}(\theta_{t+1}, p(\theta_{t+1})) - K(\theta_{t+1}) \\ &\geq \mathrm{CS}(\theta_{t+1}, p(\theta_t)) - K(\theta_t) = u(\theta_{t+1}, p(\theta_t), K(\theta_t)) \ . \end{aligned} \tag{3.4}$$

We can rewrite these three constraints as follows:

$$\mathrm{CS}(\theta_1, p(\theta_1)) \geq K(\theta_1) \tag{a}$$

$$\mathrm{CS}(\theta_2, p(\theta_2)) - \mathrm{CS}(\theta_2, p(\theta_1)) \geq K(\theta_2) - K(\theta_1) \tag{b}$$

$$\mathrm{CS}(\theta_3, p(\theta_3)) - \mathrm{CS}(\theta_3, p(\theta_2)) \geq K(\theta_3) - K(\theta_2) \tag{c}$$

From the last slide, the three constraints:

$$\mathrm{CS}(\theta_1, p(\theta_1)) \geq K(\theta_1) \quad \text{(a)}$$

$$\mathrm{CS}(\theta_2, p(\theta_2)) - \mathrm{CS}(\theta_2, p(\theta_1)) \geq K(\theta_2) - K(\theta_1) \quad \text{(b)}$$

$$\mathrm{CS}(\theta_3, p(\theta_3)) - \mathrm{CS}(\theta_3, p(\theta_2)) \geq K(\theta_3) - K(\theta_2) \quad \text{(c)}$$

Arguing as before, first increase all access fees by the same amount until the first constraint is binding. The other two constraints are unaffected.

Next increase $K(\theta_2)$ and $K(\theta_3)$ by the same amounts until constraint 2 is binding.

Finally increase $K(\theta_3)$ so that the third is binding.

Solving with all the constraints

The payoff to buying nothing is zero. This is equivalent to accepting a plan in which the use for p_0 is higher than any of the buyer's reservation prices and the access fee is zero. Given the very high price per unit, the customer consumes nothing and pays nothing.

Then we can think of the relaxed problem as one with three local downward constraints.

$$LCD(\theta_1): \quad u(\theta_1, p(\theta_1), K(\theta_1)) \geq u(\theta_1, p_0, K_0) = 0$$
$$LCD(\theta_2): \quad u(\theta_2, p(\theta_2), K(\theta_2)) \geq u(\theta_2, p(\theta_1), K(\theta_1))$$
$$LCD(\theta_2): \quad u(\theta_3, p(\theta_3), K(\theta_3)) \geq u(\theta_3, p(\theta_2), K(\theta_2))$$

As we have just argued, for the relaxed problem revenue is maximized by making all three constraints binding. That is, a type θ_t customer is indifferent between the plan designed for him $(p(\theta_t), K(\theta_t))$ and the plan designed for the immediately lower type. Economists often express indifference a follows:

$$(p(\theta_t), K(\theta_t)) \underset{\theta_t}{\sim} (p(\theta_{t-1}), K(\theta_{t-1}))$$

If type θ_t has a higher utility under $(p(\theta_t), K(\theta_t))$ then under some other plan (p, K) we say that type θ_t prefers $(p(\theta_t), K(\theta_t))$ and write

$$(p(\theta_t), K(\theta_t)) \underset{\theta_t}{\succsim} (p, K)$$

If type θ_t has a strictly higher utility under $(p(\theta_t), K(\theta_t))$ then under some other plan (p, K) we say that type θ_t strictly prefers $(p(\theta_t), K(\theta_t))$ and write

$$(p(\theta_t), K(\theta_t)) \underset{\theta_t}{\succ} (p, K)$$

The four plans are shown below. We now argue that we can replace the six question marks by appealing to Principle 2.

Note that, by Principle 1,

$p_0 > p(\theta_1) \geq p(\theta_2) \geq p(\theta_3)$

	plan 0		plan 1		plan 2		plan 3
θ_1	(p_0, K_0)	$\overset{\sim}{\theta_1}$	$(p(\theta_1), K(\theta_1))$	?	$(p(\theta_2), K(\theta_2))$	?	$(p(\theta_3), K(\theta_3))$
θ_2	(p_0, K_0)	?	$(p(\theta_1), K(\theta_1))$	$\overset{\sim}{\theta_2}$	$(p(\theta_2), K(\theta_2))$	?	$(p(\theta_3), K(\theta_3))$
θ_3	(p_0, K_0)	?	$(p(\theta_1), K(\theta_1))$	?	$(p(\theta_2), K(\theta_2))$	$\overset{\sim}{\theta_3}$	$(p(\theta_3), K(\theta_3))$

Consider the two shaded question marks. The price on plan 1 is lower than on plan 0.

For revenue maximization under the relaxed constraints, the lowest type is indifferent between the two plans. Then by Principle 2, both of the higher types prefer the higher plan.

Thus we can revise the table as shown below.

	plan 0		plan 1		plan 2		plan 3
θ_1	(p_0, K_0)	$\underset{\theta_1}{\sim}$	$(p(\theta_1), K(\theta_1))$	?	$(p(\theta_2), K(\theta_2))$	?	$(p(\theta_3), K(\theta_3))$
θ_2	(p_0, K_0)	$\underset{\theta_2}{\precsim}$	$(p(\theta_1), K(\theta_1))$	$\underset{\theta_2}{\sim}$	$(p(\theta_2), K(\theta_2))$	?	$(p(\theta_3), K(\theta_3))$
θ_3	(p_0, K_0)	$\underset{\theta_3}{\precsim}$	$(p(\theta_1), K(\theta_1))$	?	$(p(\theta_2), K(\theta_2))$	$\underset{\theta_3}{\sim}$	$(p(\theta_3), K(\theta_3))$

Class exercises:

1. Use a similar argument to replace the blue shaded question mark.
2. Then replace the pale orange question marks.

We then up with the following table

	plan 0		plan 1		plan 2		plan 3
θ_1	(p_0, K_0)	$\underset{\theta_1}{\sim}$	$(p(\theta_1), K(\theta_1))$	$\underset{\theta_1}{\succsim}$	$(p(\theta_2), K(\theta_2))$	$\underset{\theta_1}{\succsim}$	$(p(\theta_3), K(\theta_3))$
θ_2	(p_0, K_0)	$\underset{\theta_2}{\precsim}$	$(p(\theta_1), K(\theta_1))$	$\underset{\theta_2}{\sim}$	$(p(\theta_2), K(\theta_2))$	$\underset{\theta_2}{\succsim}$	$(p(\theta_3), K(\theta_3))$
θ_3	(p_0, K_0)	$\underset{\theta_3}{\precsim}$	$(p(\theta_1), K(\theta_1))$	$\underset{\theta_3}{\precsim}$	$(p(\theta_2), K(\theta_2))$	$\underset{\theta_3}{\sim}$	$(p(\theta_3), K(\theta_3))$

It follows that no customer type is better off switching to another plan. Thus all of the incentive constraints are satisfied.

While we will not consider more than three types, it is intuitively clear that the argument made above is general. Hence we have the following further principle.

Principle 5:

If the local downward constraints are all binding, that is $u(\theta_t, p(\theta_t), K(\theta_t)) = u(\theta_t, p(\theta_{t-1}), K(\theta_{t-1}))$,

then no type can gain by switching to another plan.

How many plans will be offered?

Explore with the spreadsheet

What is the intuition?

Technical Note (limitations of Solver)

Solver searches for a local maximum so use more than one starting point for plan 1

Solver stops searching for a better solution when there are no neighboring solutions that raise profit. As you will find, if you start with the user fee on plan 1 above the reservation price, i.e.

$$p(\theta_1) > a(\theta_1) \ ,$$

Solver may find a maximum by changing the other prices only. This may or not be the solution. To check, you should then start with $p(\theta_1) < a(\theta_1)$ and compare the two solutions. One may be what is called a local maximum but not the global maximum.

4. Optimal indirect price discrimination

We now argue that the monopoly can do better by offering "cell phone" plans rather than 2 part pricing plans. A cell phone plan offers a fixed number of minutes (or data) q for a total fee *r*. For a type θ_t customer the benefit from the *q* units is the area under her demand curve $B(\theta,q)$. Thus her payoff (or utility) is

$$u(\theta,q,r)=B(\theta,q)-r.$$

The benefit is the sum of the dotted and shaded areas depicted, that is,

$$B(\theta,q)=p(\theta,q)+CS(\theta,q).$$

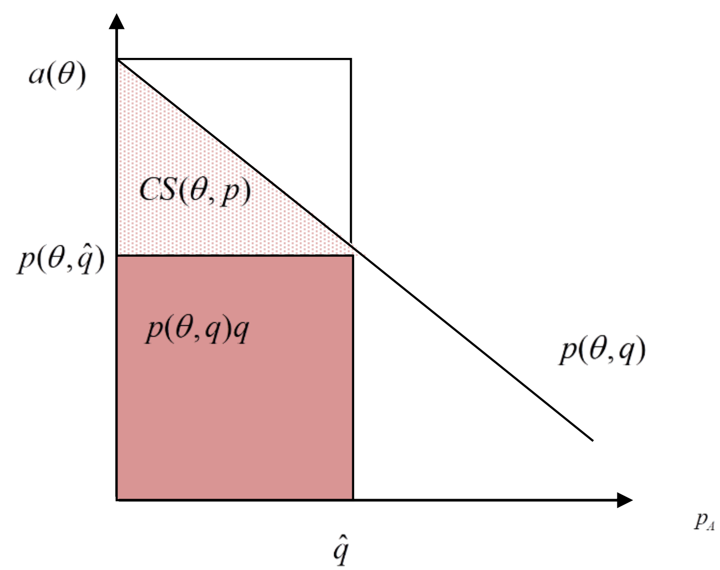


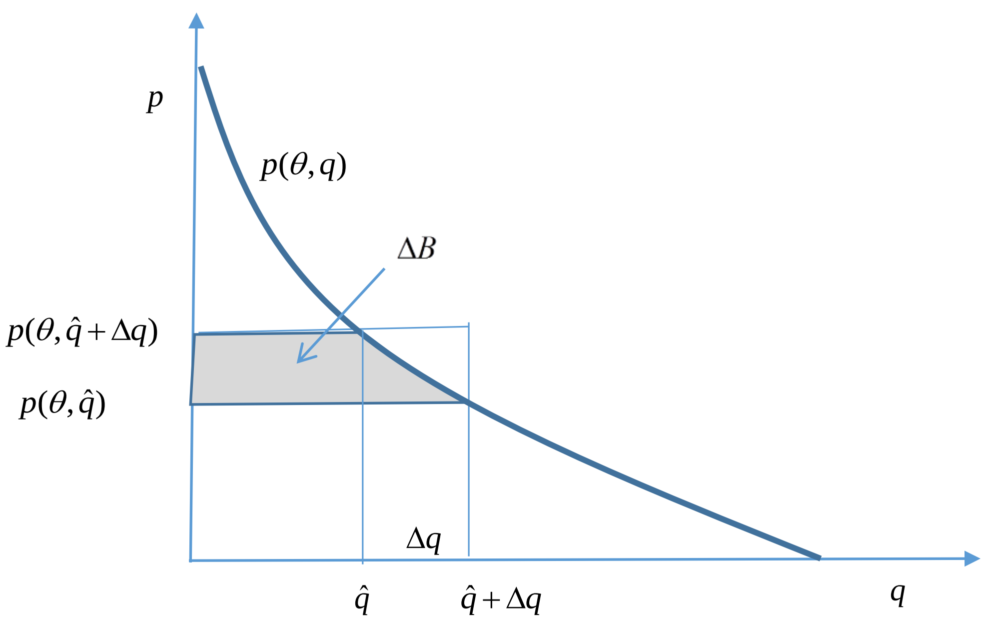
Figure 4.1: Demand price and total benefit

More generally, the extra benefit from an increase in output from $\hat{q}$ to $\hat{q}+\Delta q$ is bounded from above by $p(\theta,\hat{q})\Delta q$ and bounded from below by $p(\theta,\hat{q}+\Delta q)\Delta q$. That is

$$p(\theta,\hat{q})\Delta q \le \Delta B \le p(\theta,\hat{q}+\Delta q)\Delta q \ .$$

Then

$$p(\theta,\hat{q}) \le \frac{\Delta B}{\Delta q} \le p(\theta,\hat{q}+\Delta q)$$



Taking the limit,

$$p(\theta,\hat{q}) \le \frac{\partial B}{\partial q} \le p(\theta,\hat{q})$$

Thus the demand price function is the marginal benefit:

$$MB(\theta,q) = \frac{\partial B}{\partial q} = p(\theta,q)$$

Re-integrating, the total, benefit is the area under the consumer's demand price function.

$$B(\theta,\hat{q}) = \int_{o}^{\hat{q}} p(\theta,q)dq$$

Preferences over outcomes

Regardless of the method that a monopoly uses to extract revenue from a customer, the two things that the customer ultimately cares about are the quantity consumed and how much this costs. We call this the "outcome" for the consumer (q,r). Given such an outcome the consumer's utility is

$$u(\theta,q,r)=B(\theta,q)-r$$

It is helpful to think about the challenge for the monopoly be depicting the consumer's indifference curves over the space of outcomes, that is, all (q,r) combinations.

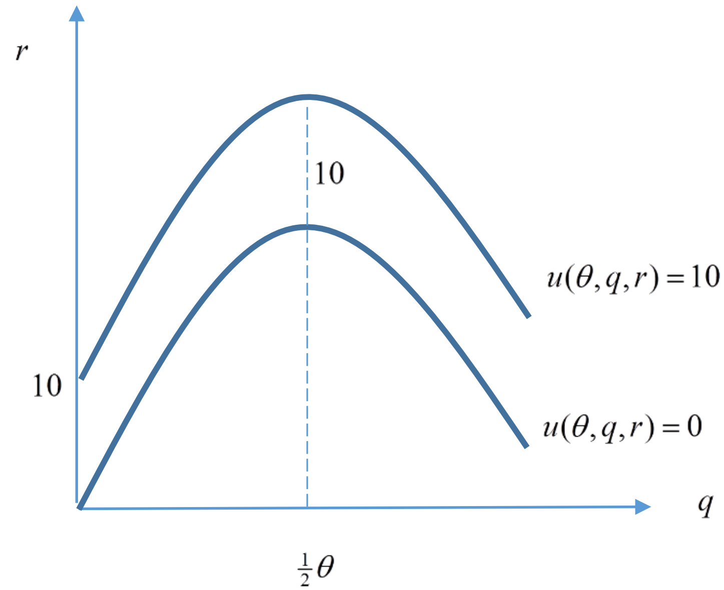
Indifference curve for a type θ customer through the outcome $(\hat{q},\hat{r})$

$$u(\theta,q,r)=B(\theta,q)-r=\hat{u}=u(\theta,\hat{q},\hat{r})$$

Rearranging yields the following equation for the total payment

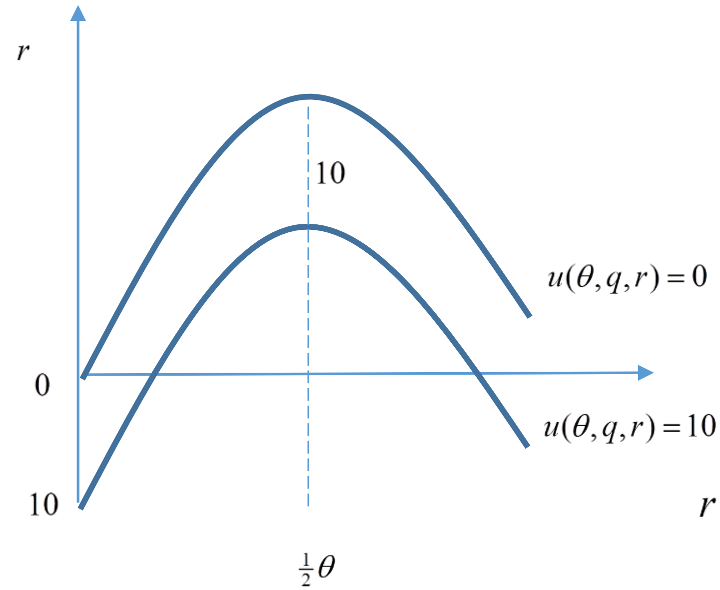
$$r=B(\theta,q)-\hat{u}$$

Note that $B(\theta,0)=0$ therefore the intersection of the indifference curve with the horizontal axis is the utility of the consumer $\hat{u}$.



To find the slope we differentiate the equation for the indifference curve.

$$\frac{dr}{dq} = \frac{\partial B}{\partial q}(\theta, q) = MB(\theta, q) = p(\theta, q)$$



Since the benefit is the integral of the demand price function, the marginal benefit is the demand price function. Note that increasing the utility holding q constant does not change the slope. (Of course r has to decline.)

Thus the gap between two indifference curves is constant.

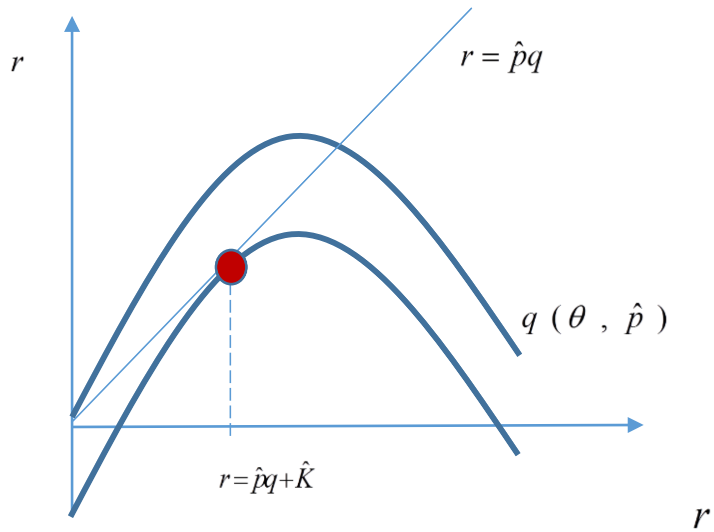
Example: Quadratic preferences

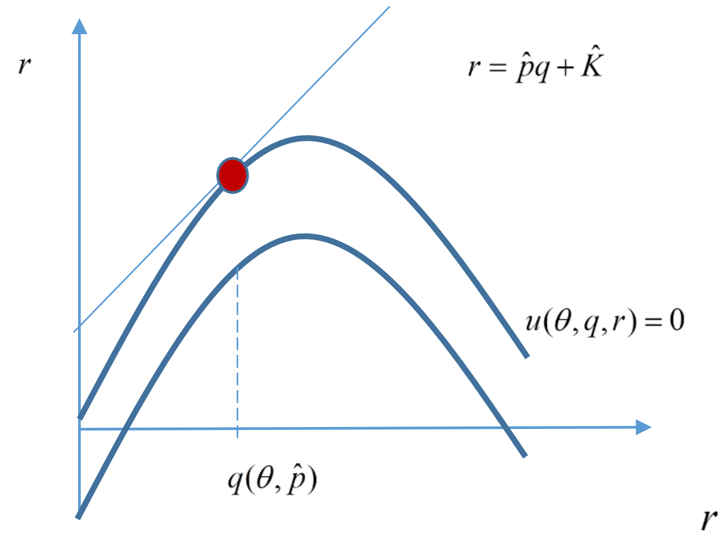
$B(\theta, q) = 2\theta q - 2q^2$, $MB(q) = 2\theta - 4q$, $u(\theta, q, r) = B(\theta, q) - r$

The slope of an indifference curve is $p(\theta, q) = 2\theta - q$.

The indifference curves $u(\theta, q, r) = 0$ and $u(\theta, q, r) = 10$ are depicted above.

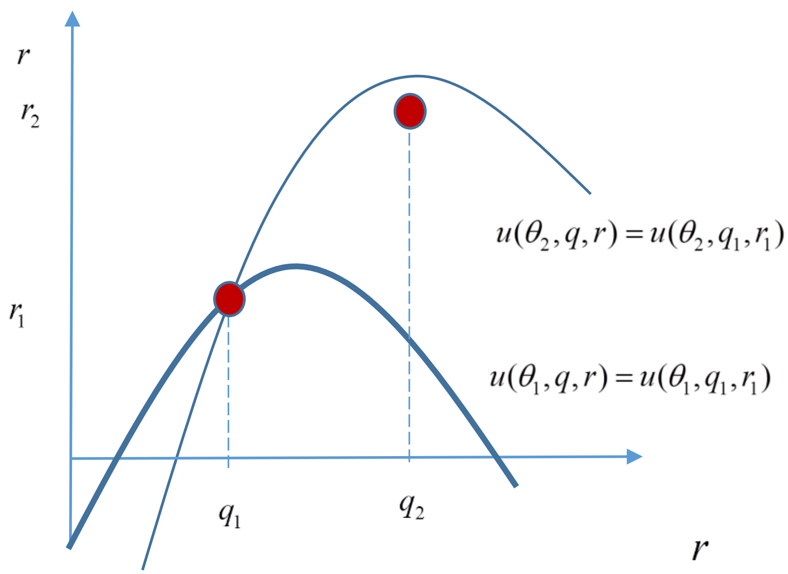
Simple monopoly **Two part pricing**





With two part pricing the total payment is $r = \hat{K} + \hat{p}q$ so the revenue line shifts upwards and extracts greater revenue.

The next figure depicts the indifference curve of a high type θ_2 with a higher demand price $p(\theta_2, q)$ and that of a low demander with lower demand price $p(\theta_1, q)$. Since the slope of each indifference curve is the consumer's demand price function. The indifference curve of the high demander is everywhere steeper.



In the figure, if the seller offers the two depicted outcomes or "plans" the two types of customer would choose different plans.

Comparison of two part pricing and optimal selling schemes

Consider the low access fee high unit price plan 1 with two-part pricing. The monopoly extracts all the consumer surplus from type θ_1 by charging an access fee $K(\theta_1)$ equal to $CS(\theta_1, p(\theta_1))$.

Now consider a type θ_2 customer who switches and purchases plan 1. Her consumer surplus is the sum of the dotted and striped areas. She also has to pay the access fee $K(\theta_1)$. However, as we have just argued, the dotted area is equal to $K(\theta_1)$. Then the payoff to a type θ_2 buyer if she switches is the striped area.

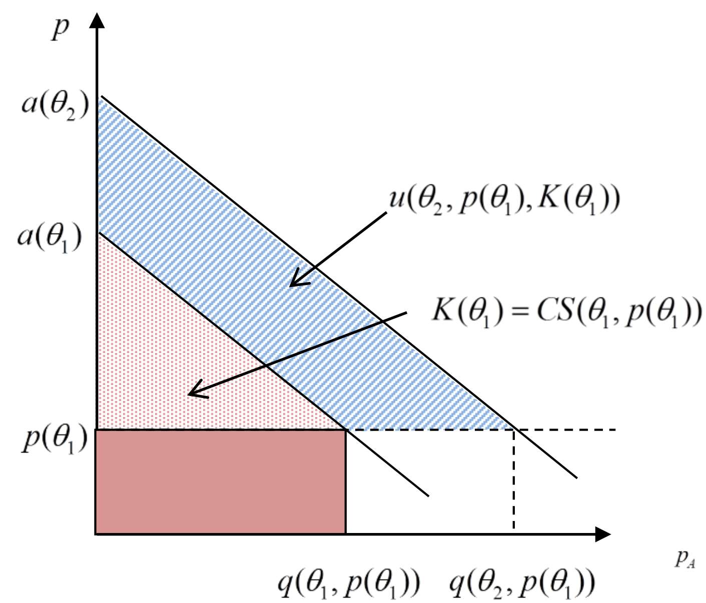


Fig. 4.2: Payoff to a type 2 buyer with 2 part pricing

Note that the total payment by type θ_1 is the sum of the shaded and dotted areas.

$r(\theta_1) = p(\theta_1) * q(\theta_1, p(\theta_1)) + K(\theta_1)$.

Suppose that the monopoly replaces plan 1

with a "cell phone" plan that offers $q(\theta_1) = q(\theta_1, p(\theta_1))$

units for a monthly payment of $r(\theta_1)$.

Same outcome for a type 1 buyer and the monopoly.

But consider a type θ_2 buyer who switches.

only gets $q(\theta_1)$ units so his consumer surplus is the

sum of the green striped and dotted areas. Since

he must pay the sum of the dotted and shaded

areas his payoff is the green striped area.

Note that this is smaller than the striped area

under two-part pricing. So switching is less attractive

and it is possible to "squeeze" a type θ_2 buyer by charging her more.

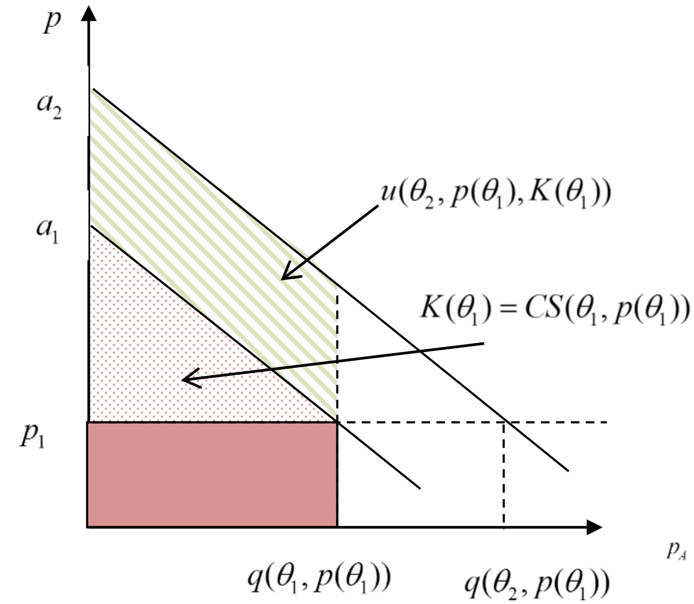


Figure 4.3: Payoff to switching declines

Suppose the firm offers "cell phone" plans rather than two part pricing prams.

$$S = \{(q_1, r_1), ..., (q_n, r_n)\}$$

A customer has the option of not participating. In this case the outcome is $(q_0, r_0) = (0,0)$. We will call this "plan 0". Then the set of alternatives available to a customer is

$$S^0 = \{(q_0, r_0), (q_a, r_a), ..., (q_n, r_n)\}$$

Let $(q(\theta_s), r(\theta_s))$ be the choice of a type θ_2 customer and define

$$C = \{(q(\theta_1), r(\theta_1)), ..., (q(\theta_T), r(\theta_T))\}$$

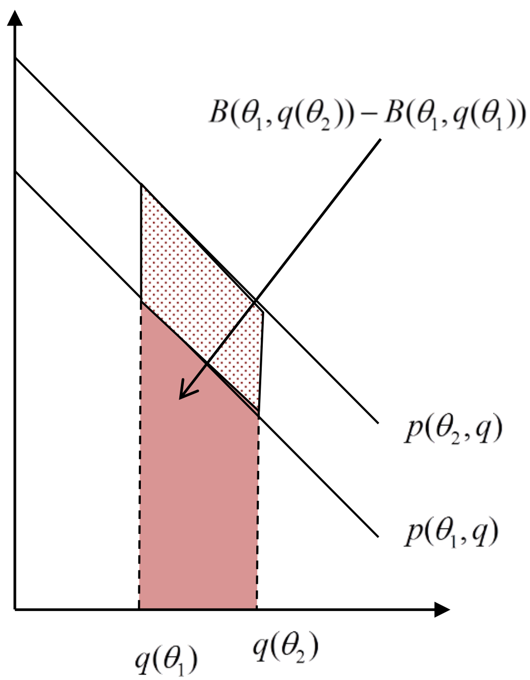
to be the set of choices of all the different types.

As with two-part pricing, we have two simple but important results.

Key to the analysis: Higher types are willing to accept a higher payment (per month) in return for a plan with a higher quantity.

Consider two plans $(q(\theta_1), r(\theta_1))$ and $(q(\theta_2), r(\theta_2))$ where $q(\theta_2) > q(\theta_1)$.

The increase in benefit if switching to the second plan is $B(\theta, q(\theta_2)) - B(\theta, q(\theta_1))$



This is depicted in the Figure. Higher types have higher demand price functions. Thus the increase in benefit is greater for higher types.

Principle $1'$: Monotone choices

Let $(q(\theta_s), r(\theta_s))$ and $(q(\theta_t), r(\theta_t))$ be the choices of type θ_s and type $\theta_t > \theta_s$.

Then $q(\theta_t) \geq q(\theta_s)$ and $r(\theta_t) \geq r(\theta_s)$

Class exercise: Consider any plan (q_A, r_A) where $q_A < q(\theta_s)$. Might a higher type choose (q_A, r_A) over $(q(\theta_s), r(\theta_s))$?

Principle $2'$:

If type θ_s is indifferent between 2 plans (q_a, r_a) and (q_b, r_b) and $q_a < q_b$ then (i) all higher types strictly prefer (q_b, r_b) and all lower types strictly prefer (q_a, r_a)

We have already established (i) in the discussion of Principle $1'$. To see that (ii) is also true we proceed in essentially the same fashion.

Profit maximizing plans

(a) Participation Constraints

The type 1 buyer cannot be forced to participate so we require that

$$U(\theta_1) = B(\theta_1, q(\theta_1)) - r(\theta_1) \geq 0 \qquad (4.1)$$

Since a higher type has a higher consumer surplus it follows that

$$B(\theta_t, q(\theta_1)) - r(\theta_1) \geq B(\theta_1, q(\theta_1)) - r(\theta_1) \geq 0$$

So if (4.1) holds, then the higher type is not being forced to participate either.

(b) Revealed preference constraints (incentive constraints)

1. Local downward constraints

No gain to switching to the plan with the closest lower quantity.

$$u(\theta_t, q(\theta_t), r(\theta_t)) = B(\theta_t, q(\theta_t)) - r(\theta_t)$$
$$\geq B(\theta_t, q(\theta_{t-1})) - r(\theta_{t-1}) = u(\theta_t, q(\theta_{t-1}), r(\theta_{t-1})) \ . \qquad (4.2)$$

2. Other downward constraints

No gain to switching to any other plan with a lower quantity.

$$u(\theta_t, q(\theta_t), r(\theta_t)) = B(\theta_t, q(\theta_t)) - r(\theta_t)$$
$$\geq B(\theta_t, q(\theta_s)) - r(\theta_s) = u(\theta_t, q(\theta_s), r(\theta_s)) \quad \text{for all } \theta_s < \theta_t$$

3. Upward constraints

No gain to switching to a plan with a higher quantity

$$u(\theta_t, q(\theta_t), r(\theta_t)) = B(\theta_t, q(\theta_t)) - r(\theta_t)$$
$$\geq B(\theta_t, q(\theta_s)) - r(\theta_s) = u(\theta_t, q(\theta_s), r(\theta_s)) \text{ for all } \theta_s > \theta_t \ .$$

Approach: Solve a "relaxed" problem

(a) Fix the prices for the plans to satisfy monotonicity:

$$q(\theta_1) \le \ldots \le q(\theta_T)$$

(b) Ignore all but the local downward constraints and choose the access fees to maximize revenue.

(c) Then show that the last constraint is satisfied.

Consider the two type case.

The constraints for the relaxed problem ((3.1) and (3.2)) can be rewritten as follows:

$$B(\theta_1, q(\theta_1)) \ge r(\theta_1)$$

$$B(\theta_2, q(\theta_2)) - B(\theta_2, q(\theta_1)) \ge r(\theta_2) - r(\theta_1)$$

If the first constraint is binding we can increase $r(\theta_1)$ and $r(\theta_2)$ by the same amount (so the second constraint still holds) until the first constraint is binding. Then increase $r(\theta_2)$ until the second constraint is binding.

Appealing to Principle 2, since type θ_2 is indifferent between the two plans, the lower type strictly prefers the plan with the smaller quantity. Thus the other revealed reference constraint is satisfied.

Principle 3: The profit maximizing monopolist chooses $r(\theta_1) = B(\theta_1, q(\theta_1))$ so that the payoff of a type θ_1 customer is zero.

Principle 4: With two types the monopolist chooses $r(\theta_2)$ so that a type θ_2 customer is indifferent between plan 2 and plan 1, that is $B(\theta_2, q(\theta_2)) - r(\theta_2) = B(\theta_2, q(\theta_1)) - r(\theta_1)$

We have therefore shown that the revenue maximizing access fees are

$$r(\theta_1) = B(\theta_1, q(\theta_1))$$

$$r(\theta_2) = r(\theta_1) + B(\theta_2, q(\theta_1)) - B(\theta_1, q(\theta_1))$$

Given these principles we can use Solver to solve for the profit maximizing plans.

The total cost of plan t is $C(\theta_t) = cq(\theta_t, p(\theta_t))$

Let $f(\theta_t)$ be the number of customers of type θ_t . Then the monopoly chooses the prices to maximizes total profit

$$U_M = f(\theta_1)[r(\theta_1) - C(\theta_1)] + f(\theta_2)[r(\theta_2) - C(\theta_2)]$$

Solving with all the constraints (Consider three types)

The payoff to buying nothing is zero. This is equivalent to accepting a plan $(q_0, r_0) = (0,0)$ in which the quantity is zero and the payment is zero.

Then we can think of the relaxed problem as one with three local downward constraints.

$$LCD(\theta_1): \quad u(\theta_1, q(\theta_1), r(\theta_1)) \geq u(\theta_1, q_0, r_0) = 0$$

$$LCD(\theta_2): \quad u(\theta_2, q(\theta_2), r(\theta_2)) \geq u(\theta_2, q(\theta_1), r(\theta_1))$$

$$LCD(\theta_2): \quad u(\theta_3, q(\theta_3), r(\theta_3)) \geq u(\theta_3, q(\theta_2), r(\theta_2))$$

Equivalently

$$LCD(\theta_1): \quad B(\theta_1, q(\theta_1)) - r(\theta_1) \geq 0$$

$$LCD(\theta_2): \quad B(\theta_2, q(\theta_2)) - r(\theta_2) \geq B(\theta_2, q(\theta_1)) - r(\theta_1)$$

$$LCD(\theta_3): \quad B(\theta_3, q(\theta_3)) - r(\theta_3) \geq B(\theta_3, q(\theta_2)) - r(\theta_2)$$

Equivalently

$$LCD(\theta_1): \quad B(\theta_1, q(\theta_1)) \geq r(\theta_1)$$

$$LCD(\theta_2): \quad B(\theta_2, q(\theta_2)) - B(\theta_2, q(\theta_1)) \geq r(\theta_2) - r(\theta_1)$$

$$LCD(\theta_3): \quad B(\theta_3, q(\theta_3)) - B(\theta_3, q(\theta_2)) \geq r(\theta_3) - r(\theta_2)$$

Step 1: Maximize revenue

$$LCD(\theta_1): \quad B(\theta_1, q(\theta_1)) \geq r(\theta_1)$$

$$LCD(\theta_2): \quad B(\theta_2, q(\theta_2)) - B(\theta_2, q(\theta_1)) \geq r(\theta_2) - r(\theta_1)$$

$$LCD(\theta_3): \quad B(\theta_3, q(\theta_3)) - B(\theta_3, q(\theta_2)) \geq r(\theta_3) - r(\theta_2)$$

Class discussion.

We will argue that for the relaxed problem revenue is maximized by making all three constraints binding. That is, a type θ_t customer is indifferent between the plan designed for him $(q(\theta_t), r(\theta_t))$ and the plan designed for the immediately lower type.

$$LCD(\theta_1): \quad B(\theta_1, q(\theta_1)) \geq r(\theta_1)$$

$$LCD(\theta_2): \quad B(\theta_2, q(\theta_2)) - B(\theta_2, q(\theta_1)) \geq r(\theta_2) - r(\theta_1)$$

$$LCD(\theta_3): \quad B(\theta_3, q(\theta_3)) - B(\theta_3, q(\theta_2)) \geq r(\theta_3) - r(\theta_2)$$

Economists often express indifference as follows:

$$(q(\theta_t), r(\theta_t)) \underset{\theta_t}{\sim} (q(\theta_{t-1}), r(\theta_{t-1}))$$

If type θ_t has a higher utility under $(q(\theta_t), r(\theta_t))$ then under some other plan (q, r) we say that type θ_t prefers $(q(\theta_t), r(\theta_t))$ and write

$$(q(\theta_t), r(\theta_t)) \underset{\theta_t}{\succsim} (q, r)$$

If type θ_t has a strictly higher utility under $(q(\theta_t), r(\theta_t))$ then under some other plan (q, r) we say that type θ_t strictly prefers $(q(\theta_t), r(\theta_t))$ and write

$$(p(\theta_t), K(\theta_t)) \underset{\theta_t}{\succ} (p, K)$$

The four plans are shown below. We now argue that we can replace the six question marks by appealing to Principle $2'$.

Note that, by Principle 1,

$$0 = q_0 \le q(\theta_1) \le q(\theta_2) \le q(\theta_3)$$

	plan 0		plan 1		plan 2		plan 3
θ_1	(q_0, r_0)	$\underset{\theta_1}{\sim}$	$(q(\theta_1), r(\theta_1))$	?	$(q(\theta_2), r(\theta_2))$	?	$(q(\theta_3), r(\theta_3))$
θ_2	(q_0, r_0)	?	$(q(\theta_1), r(\theta_1))$	$\underset{\theta_2}{\sim}$	$(q(\theta_2), r(\theta_2))$	?	$(q(\theta_3), r(\theta_3))$
θ_3	(q_0, r_0)	?	$(q(\theta_1), r(\theta_1))$	?	$(q(\theta_2), r(\theta_2))$	$\underset{\theta_3}{\sim}$	$(q(\theta_3), r(\theta_3))$

Consider the two shaded question marks. The quantity on plan 1 is higher than on plan 0.

For revenue maximization under the relaxed constraints, the lowest type is indifferent between the two plans. Then by Principle 2, both of the higher types prefer the higher plan.

Thus we can revise the table as shown below.

	plan 0		plan 1		plan 2		plan 3
θ_1	(q_0, r_0)	$\sim_{\theta_1}$	$(q(\theta_1), r(\theta_1))$	?	$(q(\theta_2), r(\theta_2))$	?	$(q(\theta_3), r(\theta_3))$
θ_2	(q_0, r_0)	$\precsim_{\theta_2}$	$(q(\theta_1), r(\theta_1))$	$\sim_{\theta_2}$	$(q(\theta_2), r(\theta_2))$	?	$(q(\theta_3), r(\theta_3))$
θ_3	(q_0, r_0)	$\precsim_{\theta_3}$	$(q(\theta_1), r(\theta_1))$	?	$(q(\theta_2), r(\theta_2))$	$\sim_{\theta_3}$	$(q(\theta_3), r(\theta_3))$

Class exercises:

3. Use a similar argument to replace the blue shaded question mark.
4. Then replace the pale orange question marks.

We then up with the following table

	plan 0		plan 1		plan 2		plan 3
θ_1	(q_0, r_0)	$\underset{\theta_1}{\sim}$	$(q(\theta_1), r(\theta_1))$	$\underset{\theta_1}{\succsim}$	$(q(\theta_2), r(\theta_2))$	$\underset{\theta_1}{\succsim}$	$(q(\theta_3), r(\theta_3))$
θ_2	(q_0, r_0)	$\underset{\theta_2}{\precsim}$	$(q(\theta_1), r(\theta_1))$	$\underset{\theta_2}{\sim}$	$(q(\theta_2), r(\theta_2))$	$\underset{\theta_2}{\succsim}$	$(q(\theta_3), r(\theta_3))$
θ_3	(q_0, r_0)	$\underset{\theta_3}{\precsim}$	$(q(\theta_1), r(\theta_1))$	$\underset{\theta_3}{\precsim}$	$(q(\theta_2), r(\theta_2))$	$\underset{\theta_3}{\sim}$	$(q(\theta_3), r(\theta_3))$

It follows that no customer type is better off switching to another plan. Thus all of the incentive constraints are satisfied.

While we will not consider more than three types, it is intuitively clear that the argument made above is general. Hence we have the following further principle.

Principle 5:

If the local downward constraints are all binding, that is $u(\theta_t, q(\theta_t), r(\theta_t)) = u(\theta_t, q(\theta_{t-1}), r(\theta_{t-1}))$, then no type can gain by switching to another plan.

Summary: Key Insights

1. Of all the participation and incentive constraints, the only ones that we need to focus on are the local downward constraints. This is because higher types are willing to pay more for a higher quantity.

2. The monopoly sets the total payments so that the local downward constraints are binding. For type θ_1 this is the constraint that he must be willing to participate.

3. By increasing the quantity sold to type θ_1 (and so increasing the benefit to type θ_1), the monopolist can increase profit until the local downward constraint is again binding. The higher $q(\theta_1)$ raises the incentive for type θ_2 to switch. Thus $r(\theta_2)$ must be reduced until type θ_2 is again indifferent. The net profit is therefore $\Delta\Pi = n_1(\Delta B_1 - c\Delta q_1) - n_2\Delta r(\theta_2)$. The monopoly sets the quantity where marginal profit is zero.

4. With more than two types the monopoly must lower the total payment on all higher quantity plans to maintain the local downward constraint. Thus marginal profit is

$$\Delta\Pi_1 = n_1 \Delta S_1 - (n_2 + ... + n_T)\Delta r_2$$

5. If the ratio $n_2 : n_1$ is sufficiently large, type 1 customers will be squeezed out of the market.

6. For the highest type, changing the quantity does not affect incentive constraints. Therefore if it is optimal to maximize social surplus and thus set $p_T(q_T) = c$