Price Discrimination

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1. Direct Price Discrimination

Consider a firm selling a product for which demand is not perfectly elastic. If the product can be easily resold, it is difficult for the firm to charge different people different prices. The firm thus exploits its monopoly power by setting a single price $p$. This is depicted below. The profit of the firm is maximized at the point where marginal revenue and marginal cost are equal.

![Figure 1: Demand price and marginal revenue](image)

How much a monopoly charges above marginal cost depends on the demand elasticity.

$$MR = \frac{d}{dq} q p(q) = p(q) + q \frac{dp}{dq} = p(1 + \frac{q dp}{p dq}) = p(1 - \frac{1}{\varepsilon}).$$

Since the monopoly equates MR and MC,

$$p(1 - \frac{1}{\varepsilon}) = MC, \text{ hence } p = \frac{MC}{1 - \frac{1}{\varepsilon}}.$$  

Differences in price elasticity lead to different prices. The bigger the price elasticity of demand function the lower is the price.
2. Direct Price Discrimination with two part pricing

For a wide range of services, resale is prohibitively expensive. A monopoly can further exploit its monopoly power by adopting a two part pricing scheme. The phone company, for example, charges a monthly access fee and a charge per minute on many plans.

Let the fixed access fee (per month, for example) be $K_t$. In addition to the access fee there is a use fee per unit $p_t$. Then if a customer purchases $q$ units her total payment is

$$R = K_t + p_t q_t.$$

A type $t$ customer has demand $q_t(p)$. In the linear case with demand price function

$$p = a_t - b_t q_t,$$

we can solve to obtain $q = (a_t - p)/b_t$. As long as $p \leq a_t$ this is the customer’s demand. For higher prices demand is zero. Therefore

$$q_t(p) = \text{MAX}(0,(a_t - p)/b_t).$$

**Profit**

Revenue from the use fee is $p_t q_t(p_t)$. We assume that unit cost is constant. Therefore the profit to the firm from the use fee $p_t q_t(p_t) - c q_t(p_t)$. This is the dotted area in the figure. In addition the monopoly charges an access fee $K_t$.

The total benefit to the consumer $B_t(q)$ is the area under the demand price function. Subtracting off the use costs yields the net gain or “consumer surplus” $CS_t(p_t)$. This is the dotted pink triangle. In the case of linear demand price functions the consumer surplus is half the rectangle, that is

$$CS_t(p_t) = \frac{1}{2} (a_t - p_t) q_t(p_t).$$
If the consumer pays an access fee of $K$, her net gain is

$$u_t(p_t, K_t) = CS_t(p_t) - K_t,$$

(2.1)

This is entered in Solver as shown below in cell H10 (see the formula bar.)

As long as this is strictly positive, the consumer is strictly better off purchasing than staying out of the market. In the limiting case where this is zero, the consumer can do no better than purchase the $q_t(p)$ units. We will assume that she does so. Then, for any price $p_t$, the monopoly maximizes profit by extracting all consumer surplus from the consumer. That is

$$U_t(p_t, K_t) = 0$$

This is entered as a constraint in Solver. The solution is shown below.
Consumer surplus is half the area of the top rectangle

\[ CS_t = (a_t - p_t) \times q_t(p_t) \]

To maximize revenue, the access fee is set equal to consumer surplus. Then the buyer’s payoff is zero.

Note that in the spreadsheet the access fee has been set too high so that a type 1 customer is better off purchasing nothing. Note also that the plans for the two types of buyer differ only in the access fee.

As long as \( p_t > c \) it is profitable to lower the use fee and so raise consumer surplus. This extra surplus is then appropriated by the monopolist by charging a higher access fee.

Note that the optimal plans differ only in the access fee. Therefore if the monopoly knows that some buyers are type 2 and others are type but does not know which, it cannot simply offer these two plans since no one will sign up for plan 2. Moreover even if the monopoly does know who are the high demanders it may be illegal to exclude high demanders from purchasing plans with lower access fees.
3. Indirect price discrimination with two part pricing

The results of the previous section hinge on three assumptions (i) resale is prohibitively costly (ii) the monopoly can identify the different types of buyer and (iii) there are no legal constraints to excluding different classes of customer from offers. Here we suppose that either assumption (ii) or assumption (iii) is not satisfied. Suppose there are $T$ different types of customer. We label them so that higher types have higher demands. Formally,

**Assumption 1: Higher types have higher demand price functions**

For all $s$ and $t > s$ if $p_s(q) \geq 0$ then $p_t(q) > p_s(q)$

For simplicity we will focus on linear demand price functions. $p_t = a_t - b_t q, \ t = 1, ..., T$ Then Assumption 1 holds, for example, if there are two types and the demand price functions are $p_1(q) = 20 - 2q$ and $p_2(q) = 40 - 3q$.

Suppose that the monopoly offers a set of alternative plans $\{(p_a, K_a), ..., (p_n, K_n)\}$. Each consumer picks one of these alternatives or purchases nothing. Note that the alternative $(p_0, K_0) = (a_r, 0)$ is equivalent to purchasing nothing. For there is no access fee and, at the price $a_r$, no consumer wishes to make a purchase. It is helpful to add this to the set of plans so that the augmented set is

$\{(p_0, K_0), (p_a, K_a), ..., (p_n, K_n)\}$

Next define $(p_s, K_s)$ to be the choice of type $s$ from this set of plans. We will write the set of choices as

$\{(p_1, K_1), ..., (p_T, K_T)\}$

Since $(p_s, K_s)$ is the choice of a type $s$ buyer,

$u_s(p_s, K_s) \geq u_s(p_t, K_t)$, for all $(p_t, K_t)$ in the set of choices.

where

$u_s(p_t, K_t) = CS_s(p_t) - K_t$

We now derive two simple but important results.
**Principle 1:** Higher types will never choose a plan with a higher use fee

To see that statement is true, let \((p_s, K_s)\) be the choice of type \(s\) and let \((p, K)\) be some other plan with a higher use fee and lower access fee.

Since type \(s\) prefers \((p_s, K_s)\) the loss in consumer surplus in switching to the other plan must be either equal or greater than the reduction in the access fee \(K_s - K\).

In the figure the loss in consumer surplus for a type \(s\) buyer is the heavily shaded area. For any higher type \(t\)

The loss in consumer surplus (the shaded and dotted areas) is greater. Thus any higher type is strictly worse off switching to the plan with the higher use fee.

**Principle 2:**

If type \(s\) is indifferent between 2 plans \((p_a, K_a)\) and \((p_b, K_b)\) and \(p_a > p_b\) then (i) all higher types strictly prefer \((p_b, K_b)\) and (ii) all lower types strictly prefer \((p_a, K_a)\).

We have already established (i) in the discussion of Principle 1. To see that (ii) is also true we proceed in essentially the same fashion.

Since type \(s\) is indifferent between the two plans the gain in consumer surplus in switching from plan \(a\) to plan \(b\) must be equal to the increase in the access fee \(K_b - K_a\).

In the figure the gain in consumer surplus for a type \(s\) buyer is the sum of
the shaded and dotted areas. For any lower type $t$ the gain in consumer surplus (the shaded area) is smaller. Thus any lower type is strictly worse off switching to the plan with the lower use fee.

**Special case: Two types**

The consumer surplus of each type if they both choose plan 1 is depicted below.

Suppose that the monopolist offers a set of plans and type 1 buyers choose $(p_1, K_1)$ while type 2 buyers choose $(p_2, K_2)$.

Suppose that a type 1 buyer’s payoff is strictly positive. Then we can raise the access fee by an equal amount $\Delta K$ on both plans until type 1 buyers have a payoff of (almost) zero. Since the cost of both plans has gone up by the same amount, type 2 buyers will not switch plans. Hence we have the following result.

**Principle 3:** The profit maximizing monopolist chooses $K_1 = CS_1(p_1)$ so that the payoff of type 1 is zero.

Note next that if a type 2 buyer chooses plan 1 her payoff is

$$u_2(p_1, K_1) = CS_2(p_1) - K_1$$

If she choose plan 2 her payoff is

$$u_2(p_2, F_2) = CS_2(p_2) - K_2.$$
Therefore the access fee for a type 2 customer can be raised until she is (almost) indifferent between the two plans.

**Principle 4:** The monopolist chooses \( K_2 \) so that a type 2 customer is indifferent between plan 2 and plan 1, that is \( u_2(p_2, K_2) = CS_2(p_2) - K_2 = u_2(p_1, K_1) \)

Given these principles we can use Solver to solve for the profit maximizing plans.

Columns A through H are exactly as with direct price discrimination. The next block of cells does the same computations if a buyer type chooses the plan intended for the type just below her. For type 1 the plan below is the option of not buying anything. We make this a plan with a very high use fee and no access fee so that the consumer purchases nothing. Cell J11 = B10 and cell K11 = C10. Columns L, M and N copy the formulas in Columns F,G and H.

There are two types of constraint. The first is the requirement that use fees be lower for higher types (Principle 1). The second is the requirement that the “local downward constraints” be satisfied with equality. That is, the blue cells in column N = the blue cells in column H.

The spreadsheet above depicts the solution. Note that a type 2 customer has a use fee equal to marginal cost and therefore consumes exactly the same as with direct price discrimination. However a type 1 customer has a use fee above marginal cost. Thus demand is lower than it would be with efficient pricing.
Below is the solution when the ratio of type 1 to type 2 customers is 2:1 rather than 1:1. Note that $p_1$ falls. This increases the consumer surplus of type 1 buyers and this allows the monopoly to extract more profit by raising its access fee $K_1$. Since the type 1 buyers are indifferent, type 2 buyers are strictly better off switching. Thus the monopoly has to lower $K_2$ to provide the incentive not to switch.

To understand these results consider the figure below.
In the left diagram the dark shaded region is revenue from the use fee less the cost of production. The dotted triangle is the consumer surplus of a type 1 customer. Since he is charged an access fee equal to his consumer surplus the profit of the firm on the type 1 customer is the sum of the shaded and dotted regions.

If a type 2 customer chooses plan 1 his consumer surplus is the area under his demand price function and above the line $p = p_1$. Since the area of the dotted triangle is the access fee, the residual striped area is $u_2(p_1, K_i)$, the payoff to a type 2 customer if he chooses plan 1. If a type 2 customer chooses plan 2 the shaded area in the right hand figure is the revenue from plan 2 less the cost of production. His consumer surplus is the sum of the dotted and striped areas. But the striped area is the minimum payoff that he must get or he will switch to plan 1. Thus the access fee is the dotted area.

Consider the right hand diagram. As long as $p_2 > c$ the monopoly can lower $p_2$ and so increase the sum of dotted and shaded areas, that is, the total profit on a type 2 customer. Then it is optimal for the monopoly to set $p_2 = c$. The revised figure is shown below.
The final step is to ask what happens as $p_1$ declines to $\hat{p}_1$. This is depicted below.

The increase in profit, $+\Delta S_1$, on each type 1 customer rises by the area in the left diagram bounded by the heavy lines. This is the increase in social surplus for a type 1 buyer. The monopoly appropriates this by raising the access fee. The reduction in profit, $-\Delta r_2$, on each type 2 customer falls by the area in the right diagram bounded by the heavy lines. The net increase in profit is then

$$\Delta \Pi = n_1 \Delta S_1 - n_2 \Delta r_2$$  \hspace{1cm} (3.1)

Note that as $p_1$ gets close to $c$ the increase in profit approaches zero while the loss is bounded away from zero. Thus to maximize profit $p_1 > c = p_2$. Note also that any change in parameter that increases the first term or decreases the second term increases the payoff to lowering the price. Thus, for example, if $n_1$ rises or $n_2$ falls, the profit maximizing use fee, $p_1$, must rise.
Solving analytically (for those so inclined)

Since \( p_i = a_i - b_i q_i \) and \( \hat{p}_i = a_i - b_i \hat{q}_i \) it follows that \( p_i - \hat{p}_i = b_i (\hat{q}_i - q_i) \). Then if \( \Delta p_i \) change in price is small, 

\[
\Delta q_i = \frac{\Delta p_i}{b_i} .
\]

To a first approximation,

\[
\Delta \Pi_1 = (p_i - c) \Delta q = \left( \frac{p_i - c}{b_i} \right) \Delta p_i .
\]

and

\[
\Delta \Pi_2 = (q_2(p_i) - q_i(p_i)) \Delta p_i .
\]

Substituting these expression into (3.1)

\[
\Delta \Pi = \left[ n_1 \left( \frac{p_i - c}{b_i} \right) - n_2 (q_2(p_i) - q_i(p_i)) \right] \Delta p_i
\]

Dividing by \( \Delta p_i \) and taking the limit,

\[
\frac{\partial \Pi}{\partial p_i} = n_1 \left( \frac{p_i - c}{b_i} \right) - n_2 (q_2(p_i) - q_i(p_i)) .
\]

This is negative if \( p_i - c \) is sufficiently small. The monopoly raises \( p_i \) until either the marginal profit from increasing \( p_i \) is zero of type 1 buyers are excluded from the market.
Three or more types of customer

The analysis of the two type case can be extended directly if there are three or more types of buyer. In Solver simply add a row for each type. Just as with 2 types each “local downward constraint is binding.

But what of all the other constraints. Consider the following table where $u_t$ is the payoff to a type $t$ customer if she chooses plan $j$.

<table>
<thead>
<tr>
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<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<tr>
<td>1</td>
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<td>=</td>
<td>u11</td>
<td>?</td>
</tr>
<tr>
<td>2</td>
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<td>?</td>
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<td>=</td>
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<td>?</td>
<td>u31</td>
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</tr>
</tbody>
</table>

To check all the incentive constraints and participation constraints we need to replace each of the question marks. Principle 4 tells us that we can do that. The price is higher on plan 0 than plan 1 and type 1 is indifferent between the two plans. Thus all the higher types weakly prefer the plan with the lower price. Next note that, since type 2 is indifferent between plan 2 and plan 1, the lower type prefers plan 1 and the higher type plan 3. Finally, since type 2 is indifferent between plan 2 and plan 3, both lower types prefer plan 2.

This yields the following table of inequalities.

<table>
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<tbody>
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<td>2</td>
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<td>u30</td>
<td>$\leq$</td>
<td>u31</td>
<td>$\leq$</td>
</tr>
</tbody>
</table>
Note that none of the three types gain by switching to another plan. Thus if the “local downward constraints” are all binding, then all the incentive and participation constraints must hold.

**Principle 5:**
If the local downward constraints are all binding, that is $U_t(p_t, K_t) = U_t(p_{t-1}, K_{t-1})$, then no type can gain by switching to another plan.

**How many plans will be offered?**

With three types it may be optimal to sell to all three types but only have two plans. For example consider the data below. Note that plans 1 and 2 are identical.

Thus there are only two different plans offered. Why is this?

**Exercise 3.1: Parametric changes**
Consider the two type case with linear demand price functions.

(a) What is the effect on the profit maximizing use fee if (i) $a_1$ rises (ii) $n_1$ rises (iii) $b_1$ falls.

(b) In which case does it follow that $F_2$ rises so type 2 customers are worse off.
Technical Note (limitations of Solver)

Consider the following example. Start with each price equal to 10 and Solver will likely give you the following answer.

Example 1

However you must also check to see if it is better to exclude type 1. Change $p_i$ to some large number and run Solver again. Here is the new solution.

Thus in this case it is optimal to have only two plans.
4. Optimal indirect price discrimination

We now argue that the monopoly can do better by offering “cell phone” plans rather than 2 part pricing plans. A cell phone plan offers a fixed number of minutes (or data) \( q \) for a total fee \( r \). For a type \( t \) customer the benefit from the \( q \) units is the area under her demand curve \( B_t(q) \). Thus her payoff (or utility) is

\[
    u_t(q,r) = B_t(q) - r.
\]

The benefit is the sum of the dotted and shaded areas below, that is \( B_t(q) = p_t(q)q + CS_t(q) \). In the linear case the consumer surplus is easily calculated since it is half the area of the rectangle.

\[
    CS_t = (a_t - p_t(q_t))q_t
\]

![Figure 4.1: Demand price and total benefit](image)

To see why this is better for the monopolist consider the lowest plan with 2 part pricing. The monopoly extracts all the consumer surplus from type 1 by charging an access fee \( K_1 \) equal to \( CS_t(p_t) \).

Now consider a type 2 customer who switches a purchases plan 1. Her consumer surplus is the sum of the dotted and striped areas. She also has to pay the access fee \( K_1 \). However, as we
have just argued, the dotted area is equal to $K_1$. Then the payoff to a type 2 buyer if she switches is the striped area.

\[ r_1 = p_1^* q_1(p_1) + K_1. \]

Suppose that the monopoly replaces plan 1 with a “cell phone” plan that offers $q_1 = q_1(p_1)$ units for a monthly payment of $r_1$. This is exactly the same outcome for a type 1 buyer and the monopoly. But with the new cell phone plan a type 2 buyer only gets $q_1$ units so his consumer surplus is the sum of the green striped and dotted areas. Since he must pay the sum of the dotted and shaded areas his payoff is the green striped area. Note that this is smaller than the striped area under 2 part pricing. So switching is less attractive and it is possible to “squeeze” a type 2 buyer by charging her more.
Rather than offering a plan in which the consumer chooses the number of units, suppose that the firm offers a fixed number of units and a total payment per month. This is the way cell phones are sold. So think of the plans as cell phone plans. The firm offers a set of alternative plans

\[ \{(q_{a}, r_{a}), \ldots, (q_{a}, r_{n})\} \]

A customer has the option of not participating. In this case the outcome is \((q_{0}, r_{0}) = (0,0)\). We will call this “plan 0”. Then the set of alternatives available to a customer is

\[ \{(q_{0}, r_{0}), (q_{a}, r_{a}), \ldots, (q_{n}, r_{n})\} \]

Let \((q, r)\) be the choice of a type \(s\) customer and define \(\{(q_{1}, r_{1}), \ldots, (q_{T}, r_{T})\}\) to be the set of choices of all the different types.

As in section 3 we have two simple but important results.
**Principle 1’:**
Higher types will never choose a plan with a lower quantity

To see that statement is true, let \((q_s, r_s)\) be the choice of type \(s\) and let \((q, r)\) be some other plan with a lower quantity and payment. Since type \(s\) prefers \((q_s, r_s)\) the loss in consumer surplus in switching to the other plan must be either equal or greater than the reduction in the access fee \(r_s - r\).

In the figure the loss in consumer surplus for a type \(s\) buyer is the dotted area. For any higher type \(t\) the loss in consumer surplus (the dotted and shaded areas) is greater. Thus any higher type is strictly worse off switching to the plan with the higher use fee.

**Proposition 2’:**
If type \(s\) is indifferent between 2 plans \((q_a, r_a)\) and \((q_b, r_b)\) and \(q_a < q_b\) then (i) all highertypes strictly prefer \((q_b, r_b)\) and all lower types strictly prefer \((q_a, r_a)\)

We have already established (i) in the discussion of Principle 1’. To see that (ii) is also true we proceed in essentially the same fashion.
Since type $s$ is indifferent between the two plans the gain in consumer surplus in switching from plan a to plan b must be equal to the increase in the payment $r_b - r_a$.

In the figure the gain in consumer surplus for a type $s$ buyer is the sum of the dotted and shaded areas. For any lower type $t$ the gain in consumer surplus (the dotted area) is smaller. Thus any lower type is strictly worse off switching to the plan with the lower use fee.

The structure of the spread sheet is almost identical to that for two part pricing. This is illustrated below.
For the same data here is the solution with 2 part pricing.

### INDIRECT PRICE DISCRIMINATION (two part pricing)

#### DATA FOR TWO TYPES

<table>
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<td>1</td>
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<td>2</td>
</tr>
</tbody>
</table>

Note that with “cell phone” pricing the monopoly is able to extract a higher payment from the buyers with a high demand and so achieve a higher profit.

### 5. Key Insights

1. Of all the participation and incentive constraints, the only ones that we need to focus on are the local downward constraints. This is because higher types are willing to pay more for a higher quantity.

2. The monopoly sets the total payments so that the local downward constraints are binding. For type 1 this is the constraint that he must be willing to participate.

3. By increasing the quantity sold to type 1 (and so increasing the benefit to type 1, the monopolist can increase profit until the local downward constraint is again binding. The higher type 1 quantity raises the incentive for type 2 to switch. Thus $r_2$ must be reduced until type 2 is again indifferent. The net profit is therefore $\Delta\Pi = n_1(\Delta B_1 - c\Delta q_1) - n_2\Delta r_2$. The monopoly sets the quantity where marginal profit is zero.

4. With more than two types the monopoly must lower the total payment on all higher quantity plans to maintain the local downward constraint. Thus marginal profit is

$$\Delta\Pi_1 = n_1\Delta S_1 - (n_2 + \ldots + n_r)\Delta r_2$$
5. If the ratio $n_2 : n_1$ is sufficiently large, type 1 customers will be squeezed out of the market.

6. For the highest type, changing the quantity does not affect incentive constraints. Therefore if it is optimal to maximize social surplus and thus set $p_T(q_T) = c$